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**Diploma in Pre School Education
(DPSE)**

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Paper – 4

**Development of Early
Mathematical Concepts in
Pre school Children**

Unit 1 : Mathematics and the Early years

1.1 :Perceptual discrimination of Quantities in infants, exact and intuitive upto 3 (subitization) and approximate discrimination of larger numbers.

Structure :1.1.1 : Introduction

- 1.1.2 :Objectives
- 1.1.3 : Research studies
- 1.1.4 : summary
- 1.1.5 :unit End questions

1.1.1: **Introduction** : Human infancy , the period of immaturity, is the longest among all living things in relation to the total life-span. At birth, the human infant is helpless than the young of most other living things. A new born calf rises its legs and suckle milk from its mother soon after the birth but the young human baby can do nothing at birth unless she is picked up and put to the mother's breast. Within a few weeks a young puppy or kitten begins to behave and act like an adult of the species. But not in case of an human being who develops and mature until he/ she is in her teens.

1.1.2 : OBJECTIVES :

After studying this unit, the student teachers will be able to

- Recognise the developmental trends in learning mathematics concept during early childhood.
- To explore the ways to facilitate children's learning of mathematics at different stages of development.

The human child is also the fastest learning creature on earth. It becomes the most complex and advanced of all. It is because she has so much to learn for which it took so long to learn. It is here that humans differ from other living things or species where humans have the ability to learn, desire to learn, and the need to learn.

It is to realize that the baby learns in the first years of life from a helpless individual with no control over her limbs. During this one year the infant learns to control her bodily movements that are enough to walk, begins to speak, communicate, needs get fulfilled, to understand her environment, manipulate using tools and many more. The learning in the first year of life is more than in any other stage of life.

Learning and development go on continuously at all times and all places and even during sleep. The period of infancy which is from birth to one year often called sensory motor stage, learns through the senses and motor activity. The five senses are sight, hearing, touch taste and smell. For an infant touch covers several aspects. It means touch not only with hands but also with the skin surface on the whole body, with the mouth, and with all the limbs. Children putting things into their mouth is part of process of exploring the world through the body. Another important sense, sometimes called Kinesthetic sense is concerned with bodily movements which is of two types : sensation which the baby experiences while rocking, swinging, being air lifted, carried, rolled; while the second one is movement which she initiates herself such as kicking, bouncing, moving the limbs first, creeping, crawling, walking, standing etc.,For young children, opportunities to learn arise from both naturally occurring, informal experiences and from planned activities. Typically occurring everyday activities provide the

stimulus for much of children's informal mathematical development. Infants, for example, learn about time and pattern through the use of rhymes and song, and develop spatial skills and awareness as they move around their environment. Likewise, the everyday activities of sharing, cooking, playing games, completing puzzles, counting, estimating distances and making music provide rich opportunities for young children to practise and develop mathematical competencies.

Perception can be defined as a process by which human infant (0-12 months age) gains awareness by responding to external stimuli. Perception is usually defined as involving the organization of the sensations (for ex, sights, sounds and smells) Cognition is what we do with those perceived sensations (for ex, classifying items into categories, solving problems memorizing).

From birth, infant actively use their perceptual systems to acquire information from their surroundings.

The study of infant perception has been the most successful endeavour in the field of cognitive development over post half century. Vision and audition senses are two senses that are developed substantially during the first year of life.

Research by Grunan et al,2001 revealed that Newborns also respond to another skin sense or touch or tactile stimulation. Research proves that preterm infant -indicates that tactile stimulation is important in ameliorating pain responses in particular and promoting normal growth and development in general.

EX: Very small preterm infants who receive extra tactile stimulation gain more weight, spends more time awake, and display more advanced cognitive and motor skills than do normally treated preterm babies. This research has led to increased prevalence of skin-to-skin care in neonatal intensive care units. The chemical senses (olfaction and taste) develop early shortly after birth.

Infants can also tell the difference among a wide range of odours early in life (steiner,1979). And develop preferences for certain odours within the first week. In a study on infants, 6-day old nursing babies were able to discriminate odours of their mothers from those of other women. Researchers found that infants develop preference for the odour of milk versus amniotic fluid by 4 days of age. 2.week old infants preferred the breast odour of a lactating female to that of non. Lactating female (making & porter 1989).

As per research studies, the infant can discriminate between mother's voice and that of another woman. By 1 week of age, babies can discriminate their mothers from another women by smell and by the sound of their voices, A number of implicit measures are taken into consideration to assess infant's perceptual abilities. Infants will alter their sucking rate to different stimuli indicating their ability to discrimination .

Researchers measure the amount of time infants spend looking at two stimuli in a visual preference parading. If infants spend more time looking at one stimulus, than another, researchers can infer infants can discriminate between the stimuli and prefer to look at one version to the other.

At about one year of age, there is a new element in the child's behaviour, The infant can establish an objective or intention in his/ her actions. At the end of infancy period, the child begins to talk using mono syllables of the language used at home, This is indicative of symbolic action.

In pre-operation school period : the child indicates objects or action, involves himself/herself in playing "lets pretend" games which are symbolic in nature involving imitation of real life character. The child engages herself in playing activities like cooking, dressing toys, modeling or drawing or in such other activity that requires imitation of the model when it is no longer present. This representation process is the transformation of actions into thoughts which facilitates in expanding the dimension of thought.

At birth, infants possess functional sensory systems-vision, (Seeing) audition, (hearing), olfaction (smell) and kinaesthetic (touch) that are fairly mature and developed. Infants learn about the world around them by experiences and gain perceptual knowledge about it. As infant grows, their senses mature, begin to coordinate information through multiple sensory modes. This process of coordination is called inter – modal perception that begins early and drastically improves over infancy.

VISION : Most basic visual functions are operational but immature at birth. The ability to distinguish fine details, "Visual acuity" is estimated to about 20/400 for most new borns. This visual acuity improves rapidly within the first few months. Colour vision, contrast sensitivity develops by four to six months.

The perception of motion is an important visual interpretation of his/her environment

People and objects move in all directions- laterally, vertically, rotational, towards and away from the observer at different velocities.

Ex : we often see that an infant responds to the rotating toy that is hung to his cradle. When the toy stops rotating, he/she responds by kicking his legs indicating that the infant wants the toy to rotate more. In other words, the infant is perceiving the motion, sound and colour of the toy. Almost all types of motion perception develop by about six months in healthy infants.

Depth perception also gradually develops during the first several months. Infants first become sensitive at about two months to kinematic, or motion-carried information for distance. At about four months, infants are able to perceive depth via the difference in the optical projections, known as stereopsis. At about seven months, infants are able to perceive depth in a flat, two-dimensional picture.

Space perception, process through which humans and other organisms become aware of the relative positions of their own bodies and objects around them. **Space perception** provides cues, such as depth and distance, that are important for **movement** and orientation to the environment. In order to reach a ball placed on the mat/rug that is out of reach for him/her, the infant tries to reach the ball by developing his own means & procedures. He finds that moving the rug moves the ball, so he pulls the rug (and the ball) towards himself.

Visual Attention: infants are born with functional eye movement system that helps them to scan the visual environment. The muscles that control the eye movement and brain- stem mechanisms that control eye muscles are fully mature at birth and hence can visualise the motion of objects and people in the environment.

Object perception : Object perception is complex involving multiple information – processing tasks such as perceiving boundaries , shapes, sizes, and substances of objects. Understanding object boundaries requires recognizing where one object ends and another object or surface begins which usually develops between three and five months in infancy. . The perception of partly occluded objects as complete is first accomplished at about two months. Newborns, despite their limited visual experience, appear to have some sense of both size and shape constancy.

Face perception :Newborns show a consistent preference for looking at faces to other stimuli throughout infancy. Infants recognize familiar faces even if there are variations in expressions and perspective , they can also recognize gender in faces. Ex : different intensities of smiling can be perceived by three months, by seven months the infants can discriminate a wide range of facial expressions like happiness, anger, surprise, sadness, fear etc although they may not be able to understand the content of these expressions. Experience with faces is thought to facilitate the development of brain areas that process facial information.

Auditory perception : Research indicates that the foetus in the womb of the mother have limited auditory experiences . Neonates auditory perception is likely to influence by prenatal experiences with sounds ,ex : they prefer to listen to the voice of their mother to voice of another woman. Infants are more sensitive to differences in sound frequency than to differences in sound intensity. Ex :Infants often like lullabies by their mothers / care takers that are soothing to their ears .

Speech perception : We often see that adults / caregivers make acoustic adjustments in their speech when interacting to infants that contains repetitions , pitch variations and simpler sentences. Infants often exhibit preferences for speech sounds that are necessary for language acquisition. over non-speech sounds; Another striking feature of infants is that in addition to speech preference, they are attentive to other forms of communication including sign language. Infants are sensitive to the patterns of rhythm and intonation in speech (prosody) by which they perceive speech , especially in bi-lingual environments.

Intermodal perception : Infants are capable of coordinating information perceived through different senses to some extent . For example, infants of two months are able to recognise their mother’s face , voice and smiles. They exhibit audio- visual relations that are presented in familiar period. Toys that produce sound and movement.

Most intermodal relations in the world are specific. Ex: speech which can be heard and seen in a talking face. Most adults sound perception is strongly influenced by watching faces and perceive the sound associated with lip movement rather than actual phoneme they heard. A five month old infant is also susceptible to this effect. By four to five months , infants may be able to recognize and discriminate objects by using information that is perceived through vision and touch.

Do you think that infants possess number sense or number knowledge ?

Infants as early as 6 months, possess an intuitive impression of numbers. A number sense that allows them to approximate magnify difference between small set of objects (Number sense is assumed to provide the basic meaning of number and quantity in infancy).

Subitize” is derived from the Latin word for “suddenly”- it is the ability to quickly identify the number of items in a small group.

“Infants begin to learn math before they sit up. They notice differences in quantity, they compare the shape and size of the objects. They use early math concepts when they play and also in other aspects of

their daily lives. Math helps children to develop the ability to think critically and solve problems. Both are integral to success in school and in life, but not all children learn the math skills they need to succeed.

Human infants in the first year of life possess an intuitive sense of number. This preverbal number sense may serve as a developmental building block for the uniquely human capacity for mathematics. In support of this idea, several studies have demonstrated that nonverbal number sense is correlated with mathematical abilities in children and adults. However, there has been no direct evidence that infant numerical abilities are related to mathematical abilities later in childhood. Here, we provide evidence that preverbal number sense in infancy predicts mathematical abilities in preschool-aged children. Numerical preference scores at 6 months of age correlated with both standardized math test scores and non symbolic number comparison scores at 3.5 years of age, suggesting that preverbal number sense facilitates the acquisition of numerical symbols and mathematical abilities. This relationship held even after controlling for general intelligence, indicating that preverbal number sense imparts a unique contribution to mathematical ability. These results validate the many prior studies purporting to show number sense in infancy and support the hypothesis that mathematics is built upon an intuitive sense of number that predates language

1.1.3 :Some of the research evidences

Fundamental skills in recognising and responding to numerical cues are apparent in infancy, and, at a very basic level, it may be innate (Wynn, 1995a; Xu, Spelke, & Goddard, 2005). Researchers studied to determine whether infants have a basic awareness of numbers. Such research, largely undertaken in psychology, has focused on demonstrating that infants, some as young as a few days old, can perceive numbers.

Researchers have argued that the human brain has specialised mechanisms for processing quantity that, at a very basic level, are innate, rather than a product of language development or culture. These basic skills are independent of language because they can be found in preverbal infants and also among animals. Investigations in developmental and cognitive psychology have aimed to identify whether infants could detect changes in numbers. Across different experimental scenarios, researchers used infants' tendency to 'habituate' to particular objects and events (that is, lose interest over time), followed by increased attention to novel objects and events (measured as increased looking time).

They found that infants can detect changes in small numbers of up to about five – this is the so-called subitising range in which the number of objects can be recognised immediately without conscious counting (Wynn, 1995b). For instance, infants (of an average age of 22 weeks) lost interest in a small number of dots (two or three) but looked significantly longer when a new array was presented (either three or two) (Starkey & Cooper, 1980). **Infants can also detect differences between large numbers.** Six-month-old infants differentiated between displays of eight and 16 objects, even when they controlled for other factors, such as area and density, which could give cues to number differences (Xu & Spelke, 2000). Later studies showed that these abilities are not limited to visual displays, with infants discriminating differences between small (vanMarle & Wynn, 2009) and large numbers of sounds (Lipton & Spelke, 2003).

Two lines of research provide evidence that children come to represent number well before the onset of verbal counting. First, infants discriminate between sets of 2 vs. 3 entities at many ages, including newborns, and with a variety of displays including visual dot displays, displays of objects with varying

properties, positions, or motions, and sequences of actions or speech sounds (Antell & Keating, 1983; Bijeljac-Babic, Bertoni & Mehler, 1993; Starkey & Cooper, 1980; Starkey, Spelke & Gelman, 1990; Strauss & Curtis, 1981; Treiber & Wilcox, 1984; van Loosbroek & Smitsman, 1990; Wynn, 1996). Secondly, infants represent that a single object added to a second occluded object results in two objects rather than one or three, and that a single object removed from an occluded two-object display leaves one object rather than two (Wynn, 1992b). Sensitivity to the effects of such additions and subtractions has been shown at various ages and with various objects in set sizes up to three.

A longitudinal study was designed to probe the relationship between infants' nascent numerical representations and the acquisition of symbolic math knowledge in early childhood. The results demonstrate that ANS acuity at 6 months of age is predictive of math achievement, number word knowledge, and ANS acuity at 3.5 years of age. Critically, these relationships hold after controlling for general intelligence in childhood. The present results suggest that symbolic arithmetic builds upon more primitive numerical representations

Two experiments investigated numerosity discrimination in 7-month-old infants, comparing their performance on numbers within the range of subitizing (2 and 3 elements) with numbers marking the limit of this range, 4 elements, or lying outside this range (5 and 6 elements). The first experiment identified 3 as the upper limit of the small range of numbers by contrasting the discrimination tasks 3 vs. 4 with 4 vs. 5. The second one found, that infants can not only discriminate 2 from 4, discrepant from the finding of Xu (2003), but also 3 from 6 elements. It is discussed, that changing the continuous quantity of small numerosities will bias the infants towards element by element comparisons instead of comparing numerosities. Nevertheless, the study corroborates the finding from Xu (2003), that there are two systems of numerosity representation.

Infants have the ability to discriminate numerical quantities in the first year of life. One knows that they can represent both the number of individuals in sets and also continuous extent.

Four-month-olds' abilities to distinguish between visual arrays containing small numbers of elements were assessed via a habituation procedure. A series of dot patterns containing either four or five elements was presented until habituation occurred. The results indicated that the infants were able to distinguish between arrays which differed only in the number of elements.

Various studies attempted to test directly whether infants represent approximate numerosities. The experiments tested for discrimination between numerosities that are too large to be handled by any mechanisms of object-based attention.

Experiment 1 tested infants' discrimination of 8 vs. 16 elements, and Experiment 2 tested infants' discrimination of 8 vs. 12 elements. Experiment 1 investigated whether 6-month-old infants can discriminate between displays of 8 vs. 16 dots that varied in size and position, under conditions that controlled for average brightness, contour length, display density, element size, and display size.

Infants distinguished between 8- and 16-element displays when continuous variables such as density of the elements and brightness of the displays were controlled. These results suggest that the "limit of 3" is not a true limit on infant's numerical competence, provided that the ratio difference between two numerosities is sufficiently large.

Experiment 2 investigated whether 6-month-old infants discriminate between large numerosities when the discrimination ratio is reduced to 1:3: a ratio that often yields success when infants are presented with small numbers (2 vs. 3) but that once led to failure with larger numbers. Infants were presented with displays of 8 and 12 elements, using the procedure and stimulus controls of Experiment 1.

Experiment 2 provided no evidence that infants discriminated between large numerosities when the difference between the two numerosities was reduced. This finding is consistent with those of Starkey and Cooper (1980), in which infants failed to discriminate four from six elements: sets that differ by the same ratio as in the present study. Together with Experiment 1, it suggests that

infants can discriminate between large sets of differing numerosity only when the ratio of difference between the sets is large.

The present experiments provide evidence that a sense of number exists in human infants by 6 months of age, at least in crude form. Infants discriminated between displays that differed in numerosity, under conditions in which discrimination could not be based on the detection of perceptual variables such as the amount of contour, average brightness, element density, or display size. Moreover, infants discriminated between sets that were too large to be represented by object-based attention.

We conclude that true representations of number, rather than representations of continuous quantities or capacity-limited mechanisms of object-based attention, underlie infants' responses. These findings complement those of a recent experiment by Wynn and Bloom (1999) on infants' enumeration of collections. As in this research, infants successfully discriminated the two-group displays from the four-group displays. This experiment, like the present studies, provides evidence for true sensitivity to numerical differences in a 1:2 ratio. Infants evidently are sensitive to numerical differences in this ratio with small set sizes as well as large ones.

The best evidence for number discrimination comes from two sources. First, experiments using the addition paradigm have compared infants' abilities to add discrete numbers of objects to their ability to add continuous quantities such as piles of sand (Huntley-Fenner & Carey, 1998) or blocks (Wynn & Chiang, 1998). Although infants successfully added solid objects with the shape, colour, and texture of sand piles or block constructions, they failed to add true piles, even though the objects and piles presented the same addition problem from the standpoint of continuous quantities and correlated perceptual variables. Second, experiments using the number-discrimination paradigm have compared infants' discrimination of two from three dots to their discrimination of four from six dots of constant size (Starkey & Cooper, 1980). Although infants successfully discriminated between the smaller set sizes, they failed to discriminate between the larger set sizes, even though both discrimination tasks presented the same correlated perceptual variables differing by the same ratio.

Starkey and Cooper argued that these findings provided evidence for a "subitizing" mechanism limited to set sizes of three or less. This argument, however, raises new problems. Faced with this problem, a number of investigators have proposed that infants' discrimination between small sets of objects does not reflect the operation of a process for representing approximate numerosity, but rather a process for keeping track of visible objects.). Because 6-month-old infants lack experience with verbal counting or formal arithmetic, the findings are consistent with the thesis that the number sense develops spontaneously in humans, as it does in other animals. Because large number discrimination is achieved only when the difference between the sets to be discriminated is large, the findings are also consistent with the thesis that the number representations found in human infants depend on a mechanism for representing approximate

but not exact numerosity, as do the mechanisms found in other animals and on Whalen et al., 1999 . When infants are presented with small numbers of objects, events, or sounds, they may attempt to keep track of each individual through mechanisms of object-based attention or other, similar devices. In these cases, infants represent each display as a collection of individual entities with distinct properties rather than as a set with a distinctive cardinality. Infants' predisposition to represent small numbers of objects or events as individuals rather than as a set may account for their preferential response to continuous perceptual variables in studies of small number discrimination: such variables characterize individual objects whereas numerosity characterizes the set but not its individual members.

. Infants' predisposition to represent large numbers of elements as a set rather than as individuals may account for their successful response to number under conditions in which continuous perceptual variables are controlled. If sensitivity to numerosity requires a 1:2 difference ratio, whereas abilities to track individual objects and events can operate on as many as three entities simultaneously, then the existence of these two mechanisms would account for the departure of infants' discrimination performance from Weber's Law.

1.1.4 :End summary

Research from developmental and cognitive psychology has yielded significant evidence that a basic capacity to respond to numerical information is present from infancy. Neuroscience has provided complementary evidence of structural location and changes in brain activity when infants detect changes in number. Such evidence shows that infants have a basic predisposition towards processing numbers, but this should not be interpreted as a fixed capacity for developing mathematical understanding. Rather, the research evidence suggests that a foundation is present very early that may provide direction for children as they grow and develop their numerical skills.

Early numeracy skills develop significantly before children are exposed to formal teaching. Research from neuroscience and psychology has been enormously influential in establishing infants' early sensitivity to number concepts. Such research, which began in the field of psychology, suggested that infants process numbers, at a very basic level, well before they develop oral language. More recent neuroscience research investigated these claims from a different perspective, and suggested unique brain activity corresponding to changes in number, or in response to incorrect calculations. Moreover, research in psychology and education describes the development of early numeracy, and suggests that skills in infancy and preschool are related to children's early success in school mathematics. In brief, the development of thinking or cognition proceeds from the perception and sensory- motor experiences to thinking in terms of manipulating concrete objects and proceeding to thinking hypothetically combining several ways of thinking in abstract terms in the absence of concrete objects.

1.1.5 :End Questions:Q1) What is perception ?How does an infant learn through perception ?

Q2) Does an infant has number discrimination skill? if so,justify with research studies.

1.2 : Early everyday experiences as basis for developing basic mathematical understanding and estimation skills.

Structure :1.2.1 : Introduction
1.2.2 : objectives
1.2.3 : summary
1.2.4 unit end questions

1.2.1 : Introduction :The development of mathematical concepts begins during early years as children participate in daily activities that involve social interactions and spontaneous play. They manipulate objects to form ideas about quantity, space and shape. Though the children are not learning mathematics formally during the early years, they are however developing mathematical concepts through various play activities like playing in the sand pit, pretend play with kitchen set, building towers, singing etc., Young children are natural learners. They construct their own knowledge, understands about quantity, relationships and symbols. They approach new tasks with curiosity and a sense of experimentation.

Research strongly indicates that young children have a strong, intuitive understanding of informal mathematics (Copley 1999; Clements & Sarama 2007) NRC 2009). Young Children have the ability to learn mathematics. Through every day experiences, they acquire a wealth of informal knowledge and strategies to deal with situations that have a mathematical dimension.

Young children have the ability to learn mathematics through everyday experiences. These experiences provide them with a wealth of informal knowledge by various means and strategies that have a mathematical dimension. These informal experiences help children to construct a solid foundation for success in school.

Children use multiple opportunities during the day to build competence in math, knowingly or unknowingly. As they pour from one container to another container, stack all big blocks in one pile, smaller ones in another, or clap to a rhythmic pattern, they are in the process of learning maths.. Everyday experiences like these provide the context for children to progress in math.

Before a child formally learns counting, “ more / less” is a word two year olds know readily. When mother gives her daughter one sweet, she stretches her hand for more.“I have three biscuits while Rani has only one biscuit”. “If I give Rani one biscuit, I will have two & Rani will also have same number of biscuits.” says Raju.

A child observing a worm or centipede says that it has many legs while they have only two. We often see children saying during the nature walk that some plants have big leaves while some plants have small leaves, that leaves turn yellow after they become old and they fall down.

Children of this early age group love to play with sand, water, clay, manipulate with clay and do models of their imagination and creativity, stack blocks of different sizes and colours much to his imagination. To most of the adults/parents, the word maths and play have absolutely nothing to do with each other. For some maths was a torture while play is something that is loved.

Children have many opportunities to enjoy and learn maths through play at home with the parents and in pre-school settings with the teacher.

Role of play-building mathematical experiences

“Play is an effective vehicle for fostering mathematical concepts and developing positive attitudes to mathematics”. The role of (curricular guidance for pre-school education) parents / adults is to extend informing the mathematical experiences at home.

Young children (3 to 8 Years) have many opportunities and mathematical experiences at home. For example they learn to count the steps at their home, start to understand

the concept of time as they become familiar with their daily routine like wash, dress, breakfast, bathing, preparation for school etc.,

-They learn about money when they go out for shopping with their parents. The daily routine also gives children practical opportunities to learn about one-one correspondence, numbers, shapes, space, sorting and matching. For example in daily routine activities , helping mother in small household chores like

-Setting plates for the family members at the dining table- -Setting plates and glasses for the family members.-one plate for mummy, one for daddy,one for sister and lastly for me. Here , though the child does not know how to count , the child arranges plates and glasses by one-to one correspondence.,Playing with water.,Pairing shirts / pants, matching socks, sorting big shirts / small shirts, sorting clothes as per the size of family members etc. Matching lids to the sauce pans.

Few play activities like these foster mathematical concepts in children.

Sand and Water play can develop mathematical concepts and language- heavy/ light, empty / full more / less etc., Conservation-how much will it hold? Shapes and patterns playing in the sand with different moulds and cups helps the child to know different shapes; Measurement. -Able to make out the differences between wet/ dry and hence compare the weights.

Home environment is the best opportunity to develop math vocabulary especially the positional words- Small, Bigger, behind, front etc.

Dough: The use of dough can help to develop mathematical understanding & use of language as short, thin, round, flat shape or 3-D shapes.

Imaginative Play: Simple activities like testing the child set the table for dinner develops counting skills, and small house-hold chores like sorting clothes for different family members, pairing and sorting socks etc.,

Books & Rhymes: Rhymes like “One-Two buckle my Shoe” develops mathematical element as well as literary skills like showing them “Print pads from left to right. Story books allow the child to count the number of animals on the page, how many things are in blue & so on.

Rhymes also develop child’s memory and sequencing which is an important higher order of cognitive skill.

Physical Play:

- Developing fine motor skills through threading of beads.
- Block play helps in sequencing as per size, colour, shape etc., increases his concentration, balancing estimation skills, understanding of weight & dimension besides developing creativity & imagination.
- Develops mathematical language first, second, how many are blue? which is biggest of these etc.,

Nature: By providing opportunities for the children to explore the nature, children develop many mathematical concepts like for eg: some plants are small while some are very big, leaves are different in size, colour & texture by planting seeds and watch them grow helps the time and life cycle of plants, measure the plants grow, compare that some plants grow fast or slow etc.,

Children are familiar with numbers because they have seen these in telephone, addresses, page numbers, clocks, calendars and so on, but the child lacks experiences with mathematical operations.

How does a teacher facilitate these informal math experiences into intentional teaching in the early years ?

The role of teacher in developing concepts is to sequentially plan number readiness programme with many fruitful activities and joyful experiences from concrete to abstract level and from simple to complex activities such as matching, identification and naming.

These cognitive skills are necessary in order to understand the concept of number. The technique of introducing the concept is to develop interest, curiosity and strengthening of cognitive skills in relation to mathematics should be made integral as a natural part of daily living in the classroom and to develop conceptual readiness in children. People use mathematics for real purposes in a variety of situations, as it is a natural part of daily living.

Classifying objects: Classification is a skill which has direct relevance for understanding the number clusters or Sets' the ability to place objects together in terms of colour, shape and size. By differentiating between various objects, children begin to think about similarities and differences and how things can be categorized.

Sorting of leaves on the basis of colour, shape & texture, sorting of seeds, pebbles, feathers etc.,

-Asking the child to sort all 2's in the box all 3's in another and so on.

Sequential Thinking: Helps the child to sequence the numerals physically and also to be able to identify the numerals. Arranging story cards in a sequence, Talking about the daily routine, Rhymes, based on sequence, pattern-making.

Problem Solving: It is the ability to solve simple day to day problems. It enhances problem solving abilities in children. Ex: Mazes, Puzzles, Riddles etc.,

Counting: Children need to give enough counting experiences which helps them to learn number names in order. Rhymes like one, two, buckle my shoe, counting different objects in the classroom, counting sounds, ordering of numbers etc.,

Developing Conceptual Readiness: Pre number concepts helps the child in making the correct assessment about the value of number before he / she understands the more complex mathematical principles. If mathematics is forcibly imposed on children before they have developed these 'Pre-number concepts they end up harming with rote learning or simple memorization. This further leads them into trouble when they are expected to apply their knowledge to advanced level of reasoning. Pre-number concepts are big/small, tall-short, heavy-light, long short, high-low, more-less, first last, left-right, & so on which should be introduced through concrete, semi-concrete and abstract examples.

Seriation cards for seriation: helps children to learn numbers in order.

One-One correspondence: Matching each object in one set with an object in the second set. This is an essential step in developing an understanding of number. It provides a basis for the acquisition of other number skills. Ex: Placing 5 leaves in a row and asking the child to place one button each under each leaf.

-To help children grasp the one to one relationship when counting .Let them wait until you touch a child's head before he is counted.

Number Concept: Number is an idea, it is an abstract concept which comes to our mind after counting things. Children who can compare the relative size of sets already to acquire some understanding of the numbers up-to 9. It helps to develop the concept of number up-to 9 in terms of their relative value.

Measuring: During pre-school years the child learns simple measurements using non-standard units- Ex: Measuring length with blocks, paper clips, fists instead of ruler. Also during early years 6 years, the child may not be able to know that the amount of liquid in a short fat cup remains the same even if the liquid is poured into a tall, skinny cup. Their perception is that taller cup has more in it because it looks like more. First & second graders may also be learning how to weigh objects using a balance scale with gram weights.

Time: Children have a basic understanding of time concepts. They realize that "Clean-up time" is in five minutes, transition activities lasts for five minutes (which is a short time). They learn at school that a year is longer than a month, a month is longer than a week etc.,

Predicting: Through experiences, children start out predicting what happens next. Ex: Because it is raining outside, outdoor games becomes an in- door activity.

Cause and effect relationship: Children can find the cause and begins to know the effects of it. Ex: It is raining and one gets wet if goes out in rain.

All these concepts may not seem "mathematical" but truly speaking they are. If a child learns to put together. The strong number-sense will enable children to understand place value of "256" that the number "256" consists of 2 groups of one hundred, five groups of 10(50) and 6 ones(6) (200) rates in elementary school. Understanding place value is necessary in learning complex addition, subtraction, multiplication and division problems. Understanding part-whole relationships is necessary while learning fractions and decimals. Problem solving is a skill that is used throughout our lives.

Therefore, young children use mathematics intuitively and develop their understanding through their individual approaches to learning as well as through their prior experiences pertaining to language, family, cultural and community backgrounds.

Teachers should provide children with planned opportunities every day to develop their mathematical understanding by incorporating high quality investigative learning experiences that involve the use of mathematics manipulation. Teachers need to provide hands on activities in the class room by carefully planning to introduce mathematical concepts, strategies and vocabulary. Children are to be provided with opportunities to explore mathematical concepts and strategies in a wide variety of ways. Opportunities by the teacher to the children are to be provided so as to reflect on and extend their understanding of maths in their everyday activities like play, conversation, rhymes etc. Children also should get easy access to variety of concrete materials, commercial products, tools and equipment so that children get to know how to use various materials to explore mathematical concepts.

1.2.3 : Summary : Math in early years (0-8 Years) set the foundation upon which future learning is built. These skills are important for being ready for school. Most children develop an understanding of addition & subtraction through everyday interactions. The young child's first interactions with his/her environment are based completely on the sensory experiences by seeing, touching hearing and also by tasting.

Many prominent Psychologists like Piaget, and Bruner, believe that perceiving through manipulation of 'concrete' objects forms the basis of human knowledge and thinking. Piaget proposed that child's thinking begins with two processes : Perception (the knowledge of objects resulting from direct contact and representation through the expression of language.

1.2.4 : Unit end questions :Q1)What are the informal play experiences you observe in a preschool child?

Q2)What is the role of play in developing math concepts in children?

Q3)what is the role of teacher in making informal play experiences into math play?.

REFERENCES :Infant Perception and Cognition

Numerosity in infants

Counting on it –Early numeracy

Unit 2 - How to make Mathematics learning enjoyable activity

UNIT 2.1 : Principles of learning mathematics –ELPS approach

Structure:

2.1.1 Introduction

2.1.2 Objectives

2.1.3 Summary

2.1.4 End Questions

2.1.5 References

2.1.1-Introduction :An engaging and encouraging climate for children’s early encounters with mathematics develops their confidence in their ability to understand and use mathematics. These positive experiences help children to develop dispositions such as curiosity, imagination, flexibility, inventiveness, and persistence, which contribute to their future success.(Clements, 2004)

Early childhood educators should actively introduce mathematical concepts, methods, and language through a variety of appropriate experiences and research-based teaching strategies. Teachers should guide children in seeing connections of ideas within mathematics as well as with other subjects, developing their mathematical knowledge throughout the day .

. Effective professional programs weave together mathematics content, pedagogy, and knowledge of child development and family relationships

2.1.2-OBJECTIVES :The student teacher will be able to

- Understand different theories of learning in ECCE
- Plan and practice developmentally appropriate practice based on the theories of learning.

A number of theorists have proposed ideas about how children learn generally, and these ideas can be related to the learning of mathematics. There are numerous theories influencing about how children can best learn mathematics.

Piaget (in Donaldson, 1978) believed that children construct their own knowledge and understanding through their interactions with their environment. This is called a constructivist theory.

Key Concepts of Piaget :logical thinking and reasoning

Schemas

A schema describes both the mental and physical actions involved in understanding and knowing. Schemas are categories of knowledge that help us to understand and interpret the world. For example, a child may have a schema about a type of animal, such as a dog. If the child ‘s experience is limited to small dogs, there is every possibility of the child to think that all dogs are small and furry.If the child happens to come across a big dog, the child will take this information thus replacing the existing schema.with new information.

Assimilation

The process of taking new information into previously existing schema, more ways of sophisticated thinking, is known as assimilation. In the above example, seeing a dog and labelling it as a dog is an example of assimilation.

Accommodation : is making observations that unseat early mis conceptions in logic.

Another part of adoption involves changing or altering our existing schemas in light of new information is known as accommodation. Accommodation involves altering existing schemas or ideas as a result of new information or new experiences. ex : the child comes to know through his experiences that there are small and big dogs with and without fur.

Equilibration

Piaget believed that all children try to strike a balance between assimilation and accommodation which is achieved through a mechanism called equilibration. As children progress through stages of cognitive behavior to account for new knowledge (accommodation) equilibration explains how children are able to move from one stage of thought into the next.

Although, each individual's thought process follows the invariant process of organisation and adaptation as equilibration of assimilation and accommodation, yet everyone is unique in his/her ways of thinking. That is because the individuals differ in their perception and representation, in the ways of processing their experiences through equilibrating their assimilation, accommodation and finally in organising the adapted bits of thinking. In each of the processes and at each stage of thinking, each child has its own and unique way.

Let us now know the different stages of cognition formulated by Piaget which provides an insight into how the child learns a concept at each stage.

Accordingly, the stages or periods of cognitive development have been categorised by Piaget as:

Sensory-motor	Period	(from	birth	to	2	years),
Pre-operation	Period	(from	2	to	7	years),
Concrete Operation Period (7 to 11 years), and Formal Operation Period (11-12 years to 14-15 years).						

Four stages of development

1. **Sensory motor:** (Birth to about age 2): This is the first stage characterised by pre-verbal and pre-symbolic period characterised by direct actions like thumb sucking, grasping, looking etc which are unco-ordinated but later becomes co-ordinated. Progression from spontaneous actions / reflex actions to acquired habits to intelligent activities. Ex : Thumb sucking activity child learns by himself, finds satisfying which Piaget calls it an intelligent act. Such habits develop out of reflex or may be conditioning from outside. During this stage, the child learns about himself and environment through motor and reflex actions. The child learns that he is separate from his environment and that aspects of his

environment – his parents, or his favourite toy – continued to exist even though they may be outside the reach of his senses.

2. **Pre-operational:** (Begins about the time the child starts to talk to about age 7): Applying his new knowledge, the child begins to use symbols or objects. Early in this stage he also personifies objects. He is now better able to think about things and events that are not immediately present. His thinking is influenced by fantasy – the way he would like things to be – and he assumes that others see situations from his viewpoint. He takes information, then changes it in his mind to fit his ideas. Teaching must take into account the child's vivid fantasies.
This is characterised as the stage of **representation** or **symbolism**. Among symbolic functions are languages, symbolic plays, the invention of fictions, and deferred imitations. During the sensory motor period, there are no use of words or symbols to represent things, and imaginations. But in pre-operation period, the girl child uses words to indicate objects or actions, involves himself/herself in playing “let's pretend” games which are symbolic in nature involving imitation of real life characters. In deferred imitation, the child engages herself in playing activities like cooking, dressing toys, modelling or drawing or in such other activity which requires imitation of the model when it is no longer present. Through such activities, representing formation is possible. Representation formation is the transformation of actions into thoughts, thus internalising overt activity which facilitates in expanding the dimensions of thought.

The preoperational thought period is devoid of reversible operations and concept of conservation. Children of four to six years of age can pour liquid from a short wide bottle into a taller thin one with the impression that they have more liquid in the taller container. Even reversing the process does not satisfy them that quantity remains the same despite one bottle being taller than the other.

3. **Concrete:** (About first grade to adolescence) The third stage, from approximately seven to eleven or twelve years of age, is that of concrete operations. It is particularly important for you, because most of the time that children are in the elementary school they are in this stage of development.

This stage marks the beginning of logico-mathematical thought and hence has importance for learning mathematics which we shall be discussing in the subsequent sections in greater detail. The child in this period begins to demonstrate actions (Piaget terms these as 'Operations') indicative of the ability to think logically by physical manipulation of concrete objects. The child is no longer dependent upon perceptions or sensory cues. During this period, the child demonstrates the two major operations of grouping and conservation which are very much associated with the development of mathematical concepts as would be clear from the discussions of the next section.

Formal Operation Period: The fourth stage is the period of formal operations which does not occur until eleven to twelve years of age. The child now in the post-primary grades, reasons hypothetically using symbols or ideas and no more is in need of physical objects as a basis for his/her thinking. The child has attained new mental structures. These new structures include the propositional combinations of symbolic logic like implications (if...then), disjunction (either-or, or both), exclusion (either – or), reciprocal implication and so on.

4. During this stage, accommodation increases, a child develops an ability to think abstractly and to make rational judgements about concrete or observable phenomena, which in the past he needed to manipulate physically to understand. In teaching, giving him the opportunity to ask questions and to explain things back to you allows him to mentally manipulate information. This stage brings cognition to its final form. This person no longer requires concrete objects to make rational judgments. At this. Point he is capable of hypothetical and deductible reasoning.

Teaching for the adolescence may be wide ranging because he will be able to consider many possibilities from many perspectives.

Vygotsky :Social interaction and learning : (in Atherton, 2011) is often referred to as a social constructivist. He emphasised the need for a child to have guidance from a ‘more knowledgeable other’ and to have opportunities to interact socially with peers as a means of learning. He also proposed the idea of the ‘Zone of Proximal Development’, which is that a child can work with someone else to achieve something that they could not achieve on their own, thereby learning through this process so that eventually they are able to perform the task by themselves. This is sometimes called scaffolding (Bruner, 1966; Wood, 1998). Similarly, Gifford (2008) refers to cumulative learning, meaning that learning needs to build on previously learnt ideas and that presenting children with something too advanced will not be effective. His theory states that the development of a student’s intelligence “results from social interaction in the world and that speech, social interaction, and co-operative activity are all important aspects of this social world” (Sutherland, 1993, p. 104).

Behaviourism attempts to explain learning through the observable interactions of the learner with the environment, without inferring anything that is going on inside the learner. It is based on the stimulus-response model of Skinner. Although the behaviourism movement has largely passed from educational research, von Glasersfeld (1995) notes “its key notions are still alive and active in the minds of many educators”

Similarly, Gifford (2008) emphasises the importance of multisensory experiences for learning. Froebel (in Beckley, 2011) also emphasised the importance of practical activities for children’s learning, including gardening and use of building blocks. This influence can be clearly seen currently in many nursery settings in international countries

There are numerous theories About how children can best learn mathematics. We all know that mathematics is an abstract subject. Children in order to undergo conceptual understanding, Pamela Liebeck in the year 1984 devised this theory of ELPS.

E- experience with physical objects

L- spoken language that explains /describes the experience

P- pictures that represent the experience

S- written symbols that generalize the experience.

Through experiences like play, touch, feel and comparison help the children in a way related to the real world. This is the fundamental part of Liebeck’s theory.

LANGUAGE : According to Liebeck’s abstract learning of maths, the next sequence is the use of language. The child is provided opportunity to explain orally the experience they had, broadens their understanding and gives scope for open discussion to consider any problems they encountered. Thus language is an important vehicle when seeking to develop mathematical concepts to children.

PICTURES : Liebeck introduced the next important stage of learning after the children are encouraged to verbally describe their experiences by incorporating pictures or diagrams as these clarify the essentials of the problem, initially. Pictures greatly assist all types of children including differently abled children. Semi concrete experiences like pictures, flash cards help to reinforce the concept in a holistic approach in understanding the concept.

SYMBOLS : the final attribute that the child needs to apply in order to fully understand the concept is the use of symbols. The children need to solve and apply their knowledge in a simplified written form

for which they need to know and recognize the appropriate symbols as per Liebeck, 1984. But it is the teacher's judgement when is the appropriate time to introduce symbols to children.

Research suggests that teaching conventional symbols at an early age can be ineffective. (Hughes 1986, Atkinson, 1992). If children are not confident enough in the use of conventional symbols, devising their own symbols provides greater understanding of the concept.

Example:step 1 : The teacher introduces big/ small prenumber concept. Initially the teacher gives experiences of Big objects like big chairs, big leaves like bamboo leaf, water melon, big ball etc., hands on experiences for small concept like small leaves(tamarind, curry leaves , mint etc)., baby chair, fruits of small size like Indian plum goose berry, insects-ants etc .

Step 2 ; After enough experiences during the process , the child is encouraged to talk about the objects that are big and small , use the vocabulary and describe.

Step 3 :The teacher gives many semi- concrete experiences in the form of pictures –big and small animals, leaves, fruits, flowers and other objects in the environment. The child is exposed to more experiences relating to the concept.

Step4 : The teacher encourages children to represent the concept in the form of symbols at a later stage. But children can adopt their own method to represent the concept. When children go to primary level ,they come to represent with symbols

Advantages : Liebeck states that children learn most effectively when the sequence of E.L.P.S. is followed in hierarchal order. Neglecting the first two stages will not help in holistic way of learning the concept while pictures and symbols alone can only engage the children for sometime. Ex: worksheets, or a text book.

Hence the stages of E.L.P.S. is detrimental to children's learning of mathematics.

The major drawback of this theory is that the teacher finds difficult in providing and using materials with the intention of creating experiences in the process of abstraction is often perceived to be ineffective .

The work of Piaget, Bruner and Liebeck all emphasises practical activity as a starting point for learning with young children and Gifford (2008) reports neuroscientific support for this approach, too. Children are naturally curious and explore the world around them. They love to play. Adults working with young children can build on this in order to support mathematical development. One way to do this is to ensure that appropriate toys and other resources are available for children to play with and for the adults to recognise the mathematical potential of these toys and resources. Adults can then observe children interacting with the resources and provide additional resources or play alongside children using appropriate vocabulary and asking appropriate questions to maximise this potential.

Skemp (1971) described two ways of understanding mathematical ideas that he called 'instrumental' and 'relational understanding'. Instrumental understanding is a shallower form of understanding. For example, we might develop an instrumental understanding of how to add, subtract, multiply and divide using a set procedure or by memorising the steps required. However, we might not understand how the procedure worked or why the various steps in the procedure are needed. One of the difficulties with this level of understanding is that if our memory of the procedure failed, we would be unable to continue. In contrast, a relational understanding of these procedures would mean that we understand how and why

the procedures work. The importance of practical activities The work of Piaget, Bruner and Liebeck all emphasises practical activity as a starting.

How to apply Bruner's Ideas in a classroom

Dr. Jerome Bruner, born in 1915, an American psychologist whose work has become especially influential in the field of education. In the book, "The Process of Education" he describes children as active-problem solvers, ready to explore new subjects and ideas and this idea has been embraced in many educational institutions world-wide.

The central idea of Bruner's work is to promote readiness of learning. In other words, shifting of focus away from external goals such as grades and motivating children through interest in a subject. The materials for learning should be visually and intellectually appealing to promote learning in them. According to Bruner, a spiral curriculum with simple to complex concepts should be adopted and to make connections between concepts.

To promote intuitive thinking by making guesses and later disapprove guesses systematically.

Ex: A meaning to a new word/concept should not be given by the teacher instantly, instead ask children to guess the meaning by looking at the words around it.

How to apply Bruner's Ideas in a classroom

The teacher has to develop connections between the concepts.

Q) What do you call this?

A. Banana

Q) What is it?

Ans: It is a fruit

What do you do with this?

Ans: We eat.

Q)What do we call the things we eat?

Ans: Food

Q) What are the other food items we eat?

This encourages children to become active learners, support discovery learning so that the teacher to tailor the material and methods for cognitive development of the child. Children initially pass through enactive stage, by tasting smelling, touching, seeing, hearing etc. Children then enter iconic stage, where they represent the world through images followed by symbolic stage where children use abstract ideas and symbols to understand new concepts. Hence discovery learning lets children pass through the 3 stages when they encounter new information.

The connectionist approach is similar to that of ELPS but differs slightly in that here teachers are not the only sources of information. It gives a broader understanding of mathematics as a whole by making connections across mathematics. It also encourages children to share their methods and the teacher adopts to suit their learning, unlike in ELPS approach. It encourages an inquisitive mind throughout child's life helping to the maximum potential of each child.

The above theories lay the foundation for learning by developmentally appropriate practice. They give a right direction for the teachers how to structure the classroom for respectful interactions with the children.

2.1.3 : Summary : The import of a learning theory concerns its implications for practice, both pedagogically in the teaching and learning of mathematics and in the teaching and learning of mathematics and in the practice of conducting Educational research. However it is the teachers' discretion to implement or wholly rule out by other elements of a learning theory. Similarly, learning theories do not imply particular research approaches. However, certain emphasis are foregrounded by different learning theories even though there are no logical consequences of them.

2.1.4 :End Questions :Q1) What are the theories of teaching mathematics ?

Q2)Which theory you as early childhood educator you would like to employ and why ?

2.1.5:References : Theories of Mathematics Education Research gate

<https://ncca.ie/media/math> Mathematics in Early Childhood and Primary Education

2.2 Nature and capabilities of the learner in early years :

Structure : 2.2.1 :Introduction

2.2.2 : Objectives

2.2.3 :Natural qualities of the child

2.2.4 :Capabilities, cognitive milestones, skills to be attained in 3-5 year children

2.2.5 Teachers role in facilitating learning in children

2.2.6 : Types of Learning, processing.

2.2.7 :Unit End summary

2.2.8: Unit End Questions

2.2.1 : Introduction : Right from birth, children are aware of their surroundings and want to explore them. As the child plays and interacts with the people and objects around him, he learns about his body, his home and the world around him. Children are natural mathematicians. They push and pull toys, stack blocks upon one another, fill empty cups with water, play with sand pouring from one vessel to

another with the scoop , digging sand and placing their feet to know the depth, etc., All these activities allow young children to experience math concepts as they begin to explore and experiment with spatial awareness, measurement and problem solving . So, children learn from their immediate environment by describing, explaining and considering the ideas.

2.2.2 :Objectives :The paper will enable the student teacher to :

- Understand the nature and capabilities of a preschool child and to plan for meaningful math activities .
- Know age-specific cognitive abilities and skills of preschool children and plan math activities accordingly.

Even routines of the day can also provide meaningful contexts for mathematical development. Often, just by listening and talking to children at these times, mathematical language and vocabulary can be developed. For example, we might talk with children about the sizes of different objects. Sometimes, the situations could be developed into problems for children to solve. Examples of questions we might ask children when laying a table are:

- ‘How many plates do we have?’
- ‘How many plates do we need?’
- ‘How many more plates do we need?’

The first of these is a simple counting question. The second could be more challenging if the people needing the plates are not all present and the child has to count from their imagination. The third question could combine the first two and involve some addition or ‘counting on’. Adults can assist children to answer these questions through modelling, by thinking aloud. Stories, songs, poetry and rhymes can also provide meaningful math experiences .

2.2.3 :Let us know the characteristics of preschool children and how these natural qualities of the child promotes cognitive development in children.

- Children between the ages of three and six years are at that stage of cognitive development when their thinking is governed by their perceptions. That is what they see before them. Ex: When five counters are placed close together and another five counters are spread apart, the child at this stage will say that the ones spread apart are more because they cover larger area.
- Their thinking is ego centric. They can see things only from their point of view. They cannot put themselves in another’s position.
- They are not yet ready for logical thinking since they do not have basic concepts.
- They are not ready for abstract thinking and reasoning.
- They learn through concrete, first-hand experience of the environment.

Here are five elements that are essential to meaningful play :

Children make their own decisions : When children choose their own play and play for themselves , they see connections between choice or results of that choice. The type of materials selected by the parents should be open ended which makes children to make more meaningful decisions. Ex : block play constructing a truck, bridge, foam wooden pieces, ribbon scraps etc are can be used as open materials.

Children are intrinsically motivated : The impulse to play comes naturally to children and thus understand the world. It is this inner drive that helps the child to regulate his/ her feelings. This self-control helps the child to success in later life.

Children become immersed in the moment : children becomes unaware of the surroundings, time and space around him while engrossed in play. Children have the security to experiment and explore, try new ideas and investigations the laws of nature . However children can realize reality versus fantasy .play.

Play is spontaneous not scripted : Very often we see that play is unplanned. The play becomes planned only if the child's toy does not co-operate or impulsively makes a change This helps the child develop flexibility and decision- making which is an important higher order cognitive skill.

Play is enjoyable: Play always has an emotional connection that is fun and joy. .Otherwise it can simply be called an “ activity”. Joy and fun results directly when the child is engaged in play.

These five essential elements of play outline the rich experiences that it provides. And children interact with others , engross in activities feel secured to try out new experiments and explore .’

Let us now know the cognitive capabilities and abilities of .the children between 3 to 6 years of age .

Childs’ understanding is determined by the extent of their experiences .They need to be provided with a variety of opportunities and experiences to foster their learning.

As children grow, they develop and acquire certain skills naturally by observations, interactions, curiosity to know- learn,ask questions etc.,These skills make their thinking expand as they grow. Ex : role play at different stages of growth. Children make sense of their information by comparing, contrasting, sorting, classification, counting, measuring, recognising pattern which promotes to think logically.and the skills they acquire.Hence they are able to organise their world conceptually for better understanding using acquired knowledge, achievement oriented tasks, resources and material available pertaining to physical and mental readiness at different developmental stages.These experiences make learning purposeful by asking questions make predictions, invent possible solutions to the problems.As they grow and understand about different objects used in daily routine they start to represent and think in complex ways, use materials to enhance their existing knowledge.

Children can learn most of the basic concepts of mathematics being engaged in activities that give them pleasure. Every child loves to play games with other children and these games that may be the perfect medium for learning several mathematical concepts. Take any familiar game children love to play and with slight modification , one can integrate some mathematics concepts in it so that the children can acquire those concepts besides enjoying the game.Early math learning is not just rote learning –drills or worksheets with one correct answer. It is playful exploration , open- ended play encourages them to solve problems in real situations. Because the

situation is real and meaningful, children get a deeper understanding of number, quantity, size, patterning and data handling.

There is no single and definite way of learning mathematics even at the earliest stage of learning. From the previous discussions in this unit, you might have formed some ideas as to the nature of learning mathematics at the early days of schooling. Many adults including teachers feel that mathematics is a serious subject and there is no place of any light hearted activity in teaching and learning process. But, for young learners at the initial stage of schooling such a serious approach invariably cause loss of interest in mathematics, developing a fear complex towards the subject and ultimately resulting in early school drop outs.

Right from birth, children are aware of their surroundings and want to explore them. As your child plays and interacts with the people and objects around him, he learns about his body, his home and the world around him.

2.2.4 : Below are some of the typical developmental milestones for “cognitive skills” (thinking and understanding).

Development of basic cognitive skills ages of 3-4 years,	Development of basic cognitive skills of 4-5 year
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|--|---|
| <ul style="list-style-type: none"> • Be able to sort and match things (e.g. recognize and match colours) • Organize things by size • Identify parts of a whole, such as the wheel of the car • Draw a very simple picture and be able to tell you what it is (you might not recognize it) • Use “why?” and “how?” questions • Tell you his full name and age • Play with an activity for a longer stretch of time (5-15 minutes) • Have a basic understanding of time, know the difference between past and present • Enjoy singing, dancing, or acting | <ul style="list-style-type: none"> • Play with words, imitating and creating sounds, and make rhymes • Point to and name many colours and shapes • Learn to identify a few letters and numbers • Draw a person with detail • Draw, name, and describe pictures • Count objects up to 5 • Tell you where he lives (street name and town/city) • Follow the rules to games, but sometimes change them as she goes • Recognize his name when he sees it printed • She is not able to tell you basic facts about herself (full name, age, city she lives in, street name, etc.) • He is not able to count 5 objects • She does not recognize her name when she sees |
|--|---|

Cognitive Development

Major Objectives	3-4 year-olds	4 - 5 year-olds	5-6- year-olds	Some Activities
II. Formation of Basic Concept				
*Formation of colour concept	*Development of ability to match, identify and name red, yellow, blue, green, white and black.	*Development of ability to identify and name more colours e.g. red, yellow, blue, green, black, brown, white, orange and purple. *Development of ability to seriate primary colours up to three levels *Development of ability to understand that mixing/blending of colours leads to formation of new colors	*Development of ability to name and seriate all colours up to five levels.	-Conversation -Rhymes and songs -Dramatization -Activities with objects, clothes, dominoes, cards, etc. -Structured play and games -Creative activities -Nature walk -primary colours -experiences, events in sequentially
*Formation of the concept of shape	*Development of ability to match and identify -Circle -Square -Triangle *Development of ability to draw a circle	*Development of ability to name the following shapes and relate these to environmental objects -Circle -Square -Triangle *Development of ability to identify other shapes, e.g. star, cone, rectangle	*Development of ability to name and draw the different shapes	Identification matching and arranging up to 3 levels.

*Formation of pre-number concept	*Development of ability to match, identify and name -Big-Small-Same as -Long-Short -Heavy-Light -Tall-Short -Fat/Thick-Thin -Wide-Narrow -Far-Near	*Development of ability to seriate up to three levels for each of the pre-number concepts- Square -Triangle	*Development of ability to seriate up to five levels for each of the pre-number concepts Identify heavy and light objects soft and hard up to 3 levels.	-Conversation and stories -Water play -Sand play -Seriation cards -Block play -Structure play and games -Dramatization -Dramatic play e.g. shop-play etc. -Structure play with objects and later number cards -Number rhymes - -Number games -Number puzzles
*Formation of number concept		*Development of ability to -understand on-to-one correspondence -form concept of numbers up to 5 - recognize the number symbols up to 5 -count and place in order numbers up to 5	*Development of ability to -form concept of numbers up to 10 -count and place in order numbers upto 10 - recognize and write number symbols up to 10	
*Formation of concept of space	*Development of ability to recognize positions like -in-out -over-under	*Development of ability to recognise and identify positions and use.	*Development of ability to indicate 'left' and 'right'	Identifying numbers and writing up to 1 to 5 and 5 to 10

Major Objectives	3-4 year-olds	4 - 5 year-olds	5-6- year-olds	Some Activities
	- front-back - above-below	appropriate vocabulary like - top-bottom -in-out -before-after -over-under -front-back-centre between -here-there -besides-behind- infront of		

*Formation of concept of time	*Development of awareness about the following concepts in terms of daily activities -day-night -afternoon-evening -before-after	*Development of awareness about the following concepts -early-late -hours as related to daily routine -time as measured with a clock -days of the week	*Development of familiarity, in a rudimentary way, of the use of a clock for reading time *Development of awareness about names of the month	-Activities with time perception cards, improvised clock, etc. -Activities related to recall of daily routine, activities in sequence
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2.2.5 How can the teacher facilitates learning of maths in classroom situation ?

Manipulation of objects: It is clear from the previous units that through the manipulation of concrete objects children acquire mathematical skills at the early stage. Acquisition of any mathematical skill at the early stage like comparison, categorization, counting, fundamental four operations, is not possible without manipulation of concrete objects. Therefore the teacher should provide variety of objects both familiar and new material and ensure that they are made available to children in the classroom for their free handling, to facilitate their learning of desired mathematical concepts.

Placing tasks in meaningful contexts: In real situations, where the mathematics serves real purposes, young children quickly and easily develop their own informal and largely effective methods. Evidence from research about the ways children learn seem to suggest that what we need to do is to start with real problems, and work from them towards abstract representations. There are abundant opportunities in the everyday activities of young children to get themselves involved in mathematics. Playing games, sharing sweets, groupings in the class for performing different activities, finding out the number of days next to a school holiday, counting the number of red and yellow blocks in the structure they have made are just a few such examples. With young children in particular, problems can be real yet essentially born out of imagination. Use of fairy tales, adventure stories, comic strips are some of the examples of materials that can spark imagination in children. By providing real problems that are placed in meaningful contexts, they tend to become less dependent on such context and children be given confidence for drawing abstractions from real and concrete experiences. The teacher can provide intentional opportunities for problem solving by posing questions pertaining to stories such as “if you were a pigeon, how would you help the ant from drowning? or questions that help in think and answer or to a solution.” How would you reach home because it is raining outside? “ or the teacher very commonly asks question that how we would share a pencil among three children who forgot to bring.

Representation in multiple ways: Another important element which is required to help children move towards abstract thinking in mathematics involves helping them to develop their representational abilities as perceived by each child. It is now an established fact that children should be given opportunities to make their own representations of mathematical problems, processes and procedures before they are introduced to the conventional symbols. It is clear that if children are to become able and confident in solving mathematics problems, they must be able to represent mathematics and to others and themselves in language and in mathematical symbols. Many mathematics educators now believe that it is important that children express their mathematical thinking in language, through talk, before they begin to represent it on paper and before they use mathematical symbols

- ✓ This involves children in doing mathematics practically, and then following a five-step sequence of activities towards recording as follows:
- ✓ The learners explain their thinking to others;
- ✓ They demonstrate their mental images either with objects or by sketches/illustrations;
- ✓ They record in writing the 'story' of what their sketches show; They make successive abbreviations of the process they used; Finally, they can see the relevance of and adopt standard notations.

Developing alternative strategies: When the children can form representations, they can also develop ways to calculate and solve mathematics problems other than the prescribed ways given in the textbooks. That a child can evolve his/her own method of calculation, stems from the observations of totally non-schooled children performing calculations of various types required in their daily lives which are different from those given in textbooks. This lack of a relationship between informal and formal methods is a major cause of young children's loss of confidence with school mathematics. Developing new strategy may not be always possible for children. But, whenever any child comes up with any new one, he/she needs to be reinforced. Searching for alternate strategy need to be a regular feature in the classroom transaction. After the discussion of any operation or procedure for solution of a problem, children may be encouraged to think of any alternate strategy of the one discussed in the class either individually or in groups. Very often, the mathematics teacher is very rigid about the formal methods given in the mathematics textbooks and does not allow any slight deviation from those. Such an attitude does not help children to explore alternative strategies and loses interest as a teacher in mathematics, need to recognize the ability of the children to build alternative strategies and encourage it as much as possible.

Problem solving and problem posing: Solving mathematics problems and the process of problem solving, although are different, have a lot of similarity in understanding the problem, suggesting and trying out different possible procedures of solution and solving the problem. Problem solving abilities can be developed when we encourage children in solving the problems independently or in groups without providing any direct support. Besides promoting problem solving abilities in children, they should be encouraged to pose problems. Posing relevant problems indicates the level of understanding of the concepts, processes and procedures of mathematics. You should encourage such practices in the classroom as much and as frequently as possible.

2.2.6 :From psychological point of view, the dominant model of human learning is of the child as an information processor, attempting to derive meaning from experience by subjecting it to several modes of processing like classification or categorisation of new information and relating those to the existing experiences to derive meaning. There are three main features of the *human information processing system* which have very direct implications for introducing young children to the world of formal mathematics.

1. **Learning by induction:** Inductive reasoning is the basic process whereby children can easily make sense of their world by classifying and categorising experience into increasingly structured conceptual structures and models. The overwhelming significance of inductive process for children's learning has long been recognised, and has long been a strong element.

2. **Limited 'working memory' capacity:** While teaching mathematics we usually are not aware that human being has a limited capacity of processing information. For example, Miller has demonstrated from a whole range of evidence that we hold only about seven separate pieces of information in our short-term or 'working' memory. This is why as adults, we can easily process a sum such as 17×9 , but have much greater difficulty with 184×596 to do it mentally. We know the second procedure we must go through to get the answer to the second sum, and we can carry out each of the separate computations involved.

173. Development of 'meta-cognitive' awareness and control: The third general feature of the human processing system which we must consider is that it is a system which not only learns but learns how to learn. One is aware of ones'own ways of thinking or learning, what the American psychologist has termed 'meta-cognition'; he/she acquires more ability to control over his/her actions and learning. As a teacher, one has to focus on finding multiple ways of solving by exploring his/ her meta cognitive skills rather than mere seeking solutions from the text book.

Young children need to see themselves as capable mathematicians. Child-guided and child-focused explorations and teacher-guided math activities help children practice and consolidate their learning. This helps them feel confident about what they know and can do. Although many preschoolers learn some math concepts on their own, it's important for teachers to include math in authentic experiences, resulting in a deeper understanding by children (ETFO 2010).

In addition to creating a rich math and manipulatives learning center, teachers can encourage children to use math tools and strategies in all areas of the classroom. Children might use a set of plastic links to measure their buildings in the block center, use play money to pay for a train ticket in the dramatic play center, and use rulers to measure the growth of spring bulbs in the discovering science center. Take a set of scales outdoors so children can figure out who found the heaviest rock. Using math tools for real-life tasks frees both teachers and children to act spontaneously, resulting in richer interactions and a calmer learning environment (Wien 2004).

In addition to the freedom to use materials in authentic ways, children also need freedom of time and space to deeply engage in math. The preschool schedule should include plenty of time for uninterrupted play so children have the time they need to work on sustained tasks of interest. This allows children to explore materials thoroughly, often resulting in more complex and evolved experiences over time. If a child spends all of his time at one learning center, he is not missing out on learning opportunities elsewhere. Instead, his deep connection to the center is often indicative of rich learning. Teachers can model the use of other materials at the center, such as using writing materials to draw plans for a structure to be built, or pose challenges that encourage the child to think beyond her play, such as *How tall can you build this tower before it falls?*

To support learning, it is important to encourage children to communicate their explorations and findings. Teachers can establish a routine through which children share their experiences at group time. For example, a child might explain how he built a structure with blocks, do a dance with repeating steps, or share a photo of a complex pattern made with colourful buttons. While circulating through the room, a teacher might notice high-quality work and suggest that a child share it with her peers during group time. The child making the presentation grows in confidence and the onlookers may want to try the experience themselves.

2.2.7 :Summary: Unaware of the information processing abilities of children, the teachers and parents drive the children to perform better in mathematics by sheer memorising and repeating the processes taught to them, predominantly deductive in nature, without developing adequate understanding and interest in the problems given in mathematics textbooks which have little relevance to their real life problems. . Early childhood mathematics is vitally important for young children's present and future educational success. Research demonstrates that virtually all young children have the capability to learn and become competent in mathematics. Furthermore, young children enjoy their early informal experiences with mathematics. Unfortunately, many children's potential in mathematics is not fully realized, especially those children who are economically disadvantaged. This is due, in part, to a lack of opportunities to learn mathematics in early childhood settings or through everyday experiences in the home and in their communities. Improvements in early childhood mathematics education can provide young children with the foundation for school success.

2.2.8 : Unit end questions :

Q1) What are the characteristics of learning in preschool children?

Q2)What are the basic cognitive skills you observe in a four year child?

Q3)what is the role of teacher in promoting cognitive concepts in children?

2.3 Linking Learning with purposeful and meaningful experiences :

Structure : 2.3.1 :Introduction

2.3.2 : Objectives

2.3.3 : Number sense and spatial ability in children

2.3.4 Maths in -:Choice time activities, block play, dramatization, music,

2.3.5 ;Block play, sand play and outdoor play mathematics

2.3.6 :Five components of mathematics

2.3.7 : End summary

2.3.8 Unit end questions

2.3.9 :References

2.3.1 : **Introduction** : Most young children enter school having already been involved in a range of mathematical experiences, most of which occur naturally during their day. Mobile infants explore spatial relationships as they navigate the space around them. They crawl and climb over, under and around objects, and then they position themselves so they can reach a toy. Toddlers learn about shape and size as they explore blocks, stack them, and knock them down. Two- and 3-year-old children help family members sort laundry, cook, and set the table. Such activities involve sorting, matching, counting, and measuring.

Preschool teachers build on these experiences. They can provide children with meaningful opportunities to count; measure, collect data; and explore space, shapes, and patterns. They can nurture reasoning, problem solving, communicating, and representing all day long. Of course, some of these occasions arise unexpectedly, for example, when a child announces that she found an ant hill with “a million ants.” However, chance occurrences are not sufficient; teachers must purposefully incorporate mathematical experiences in each event of the day.

2.3.2 : **OBJECTIVES** : The paper will enable the student teacher to

- Understand how every day experiences and social interactions in an activity form the basis for mathematical thinking in children.
- Make use of childrens’ spontaneous play activities as well as specific play activities to foster development of mathematical concepts.
- Realize the importance of informal experiences and to make into meaningful experiences.
- **To** understand the role of teacher in promoting mathematical concepts in children
- **To** know the importance of joyful learning in teaching-learning process.

Children show their number sense in many everyday problem-solving situations involving numbers and measurement. They reason out who has more or less, devise strategies for creating equal shares of countable objects or amounts, or use counting in a range of situations to reason about a single group of objects or to compare two groups. Children informally build these skills in their everyday interactions with parents or caregivers or with other children, and with other children, and they can be encouraged to develop their understanding in play situations or informal situations. Counting was considered as a rote activity in early years but now researchers consider it as an important emerging conceptual development which lays foundation for later mathematical complex skills.

Childrens’ counting skills should be learnt accurately through

- Developing a stable counting sequence
- Learning to count one item at one time (one-one correspondence)
- Knowing that the last number counted is the number of objects in the set (cardinality)

In two year old children we find counting objects in a stable sequence is well developed while children between two and four old children rarely make mistakes in one to one correspondence counting of small set of objects. Preschoolers understanding of cardinality develops as the child grows and matures. Children who develop mastery over counting skills enables them to solve increasingly complex problems- uses as a problem solving tool to use counting to compare the size of two sets, count accurately without physically touching the objects, use counting to solve addition and subtraction problems, learns to count from the larger set. Children of two years of age quite reasonably identifies the larger of two sets of objects three compared with four, three compared with five etc, though their counting skill is not developed fully. Five year old children show estimating skills of identifying greater of two large quantities. This understanding of simple concept of more/less is acquired before they can use counting to compare numbers. We often see children sharing one to one when mother gives them puffed rice to eat., which is a strategy for equal shares. Childrens ' informal number sense provides a foundation for their school mathematics achievement and strongly predicts their mathematics competence later in school. (Geary 2015).

Many recent longitudinal studies have investigated mathematical development during transition from preschool to early primary. Counting skills and understanding in early primary predicted later math achievement one year later.

Research now proved that the emergence of counting skills is an important conceptual development.

Griffin in 2004 has argued for specific number sense content for a two year old and suggested the following steps in development of number sense.

- Knowing numbers indicate quantity and thus have a magnitude .
- Understanding and using relative terms such as more/ , less, bigger/ smaller
- Knowing numbers in the counting sequence have a fixed position.
- Understanding the sequence of numbers , ex; three comes before four
- Knowing higher numbers reflect greater quantities ex :four is greater than three.
- Knowing each count term represents a unit increase.

2.3.3:What is Spatial Ability?

Spatial ability is the comprehension and recall of the spatial relations between objects. This ability is seen as a specific type of intelligence along with logical reasoning, verbal aptitude, and memory skills. Without fully developed spatial awareness, humans would be unable to feed themselves, play games and sports, or generally move around and function without crashing into everything in sight. Well-developed spatial awareness is also linked to successful skills in reading and math as well as to artistic creativity .

We use spatial skills in our everyday life, for example, when we travel park, navigate with a map, catch a ball, or arrange furniture. Spatial ability is more intensively utilized in math class, for example, understanding a number line, finding geometric patterns, or determining whether an object is vertically or horizontally symmetrical. research suggests that spatial thinking is an important predictor of achievement in STEM, or science, technology, engineering and mathematics.

Spatial reasoning is strongly connected to math skills.

For example, children's quality of block play at age four has been found to predict their math achievement in high school. Also, when researchers directed fourth graders' attention to the symmetry between positive and negative numbers (that -3 is the same distance from zero on a number line as + 3, for example), the students got better at solving problems in that area – and in higher level math they hadn't seen before.

Spatial skills have been found to explain at least part of the difference in math grades between girls and boys. Traditionally, boys have played more with toys like blocks and construction sets than girls. Spatial orientation As preschoolers learn to identify objects, they need to use spatial orientation vocabulary to describe the relative positions of the object. It is therefore necessary for the preschoolers to understand and use vocabulary regarding positional objects such as above, below, top, below over/ under, inside/ outside, front/ back etc. The preschool teacher has to intentionally use the positional words as given below :

- **Focus on a word a week.** The positional words should be used in mother tongue, throughout the day in the classroom, in the playground etc., Ex; You are sitting beside your brothers' friend. , Oh, your ball has gone under the cot. How do you reach the ball. ?You cannot see the bottle, it is at the back of the bag..
- **Pair positional and shape vocabulary :** It is a round photoframe which is beside the box. .
- **Create a book.** Invite families to write about a favourite activity using positional words. We drove next to the park, travelled under the expressway, and walked over the bridge. Children can provide the illustrations.
- **Use photo examples.** Take photos of the children demonstrating positional concepts. Raju is standing near the wall piece. Add photos and words to the word wall.
- **Play spatial Ramu Says.** Give each child in a small group a stuffed animal and play Simon Says using positional vocabulary. Ramu says put your animal above your head. Put your animal under your chair.
- **Create positional obstacle courses.** Encourage preschoolers' use of positional words during play or transitions. Before going outside, climb up the steps, slide down the slide, jump over the cones, and line up next to the door.
- **Narrate actions with orientation.** Use positional words to describe how the children move from one place to another. You parked your tricycle in the playground and next to the window. You walked under the skylight and over the carpet to enter the classroom.

Preschool teachers can create environments and plan activities so young children and their families are both enveloped and engaged in mathematics. Programs can feature numeric and geometric representations with appropriate vocabulary terms. Beyond the classroom, preschoolers can discover and enjoy mathematics in their homes and communities.also.

There is a feeling among large sections of adults including teachers that mathematics is a serious subject and should be taught with all seriousness and there is no place of any light hearted activity in teaching and learning process. But, for young learners at the initial stage of schooling such a serious approach invariably causes loss of interest in mathematics, developing a fear complex towards the subject and ultimately resulted in early drop out from schools. Children can learn most of the basic concepts of mathematics being engaged in activities that give them pleasure. Every child loves to play games with other children and these games that may be the perfect medium for learning several mathematical concepts. You can take any familiar game children love to play and with slight modification you can integrate some mathematics concepts in it so that the children can acquire those concepts while

enjoying the game. In addition to these efforts, you can devise interesting activities specifically.

How can you prepare a preschool classroom a math rich environment ?

2.3.4 :Choice time activities

Preparing the environment

- Create choice boards for interest areas that indicate the number of children who may be in a given area at a time.
- Materials should be organised in ways that encourage children to sort and classify.
- Equip all interest areas with mathematics materials particularly dramatic play.
- Math –related books appropriate to each interest area should be added.
- writing, drawing, and construction materials are to be added sothat children can represent their discoveries and learning.
- Post step- by- step instructions for using equipment (e.g., computers) or handling routine tasks (e.g., cleaning up an area).
- Time can be fixed for a particular activity so that children can learn about and mange their time at a favorite interest area or with a new or favorite toy or game.

Small –Group time

Preparing the environment

- Prepare print or visual materials (e.g., recipes, song charts, and rhymes).
- Add props, flannel materials, and storyboards for storytelling or retelling.
- Prepare materials and supplies such as manipulative, tapes/cds, games, or writing materials to conduct a focused activity.

2.3.5 :Mathematics in the block area

The block area is a favourite activity among children and presents children with numerous opportunities to explore mathematical concepts such as shape, size, space, pattern, and number. Children also acquire and refine mathematical process skills as they address problems such as how to steady a tower so that it does not topple, how to construct a bridge over a raging. river, or how to make the roof stay up. Children can persist in solving problems when they are fully engaged in their block building activity.

Role of teacher : :teachers should first observe children in the block area .After carefully chosen questions , math vocabulary be introduced, nurture childrens' understandingof mathematical concepts and support their development as serious builders, problem- solvers, players etc .,ex: how many blocks did you use for the tall tower you made? Can you build a road with these blocks ?(number sense); Can you give that rectangular one? I am searching for curved blocks that fits here.(geometry and spatial sense) ; Your road is how many blocks length ? can you tell me?(measurement) ; the road or floor you

made is specific design, can you explain?(pattern); you have put all yellow blocks in one pile and blue ones in another pile.(data analysis)

Childrens' progress of skills :observe children for the indications below.

Counting blocks, one to one correspondence, talking with peers /adults describing what they have built/, matching shapes, solving problems by balancing one over the other, constructing structures, naming the shape of the block, arranging in a sequence etc.,

Mathematics in the dramatic play area

In the dramatic play area, children experience mathematics in ways similar to their mathematical experiences at home and in their communities. Enacting real-life situations such as setting the table, sorting, laundry, or shopping for shoes allows them to experience mathematics in meaningful ways and to deepen their understanding of everyday experiences. They also have opportunities to solve real problems like determining how to divide a pizza equally among friends. Children's ability to engage in dramatic play, to imagine and recreate past situations, or to substitute one toy for another provides practice with abstract thinking and using the symbols of mathematics.

The teacher should observe children engaged in dramatic play and thoughtfully promote mathematics learning in children. For ex :lunch is ready , latha, ravi and aruna., how many places do you need to set the table? (number and operations), Where will we arrange the bed for the baby?(spatial sense), I don't think the dress will fit the doll, what do you say?(measurement), I like watermelons, did you observe the dark green, light green colour patterns on it?(patterns), I see that you are sorting clothes of family members,(data analysis) etc

The teacher has to look for these indicators during dramatic play : placing objects in one to one correspondence, counting, measuring, using ordinal numbers, , sorting, comparing sizes, using positional words, solving problems etc.

Using sand and water to teach mathematics

Teachers can take advantage of children's natural in and enjoyment of sand and water to introduce the language of mathematics and teach many concepts and skills. Encourage children to talk about and explain what they are doing e.g., "Tell me how you made this tunnel." Supply words, e.g., "It's full. Now it's half-full. Now it's empty." Describe what children are doing e.g., "You are splashing water over the side of the water table." Wonder aloud, e.g., " What do you think?" Ask questions, like "which one is heavier? Why do you think that is?"

Some children enjoy using sand and water for pretend purposes, e.g., " I added sugar; now it's lemonade." The teacher can can support their creative play through his/her responses and expression of interest. Because the teacher recognizes that free exploration is valuable for children, need to observe children for a while before stepping in to promote their understanding of mathematical concepts and vocabulary.

Questions and comments can help focus children's attention on the mathematics involved in sand and water play, but care should be taken that questioning can also interrupt children's thinking and distract

their learning. Always begin by observing. Then thoughtfully determine whether, when, and how you should step in.

Observing children's progress

As children explore sand and water, look for these indications of mathematics learning.

- Estimating and counting (e.g., the number of scoops to fill a pail)
- Comparing capacity (e.g., using terms like some, all, more, the same amount, less, too much, not enough, left over)
- Recognizing sizes(e.g., tall, taller, tallest; wide, wider, widest; biggest, lowest; thin, narrow; shallow, deep)
- Comparing mass (e.g., heavier, lighter) and speed (fast, faster, fastest; slow, slower, slowest)
- Using nonstandard measurements (e.g, tubful, bowlful)
- Sorting and classifying collections of objects
- Describing shapes(e.g., round, curved, straight)
- Using positional words(e.g., around ,on top of, over, under, through)

Mathematics in the music and movement area

Music and movements activities can enhance children's understanding of mathematical concepts. As children clap and dance to the beat of the music, they experience patterns physically. Moving to the tempo, or speed, of the music enables them to experience time, a measurement concept, firsthand. Children explore geometrical and spatial concepts as they transform their bodies into a ball and move forward and backward, around and through, back and forth, or up and down. Counting is reinforced by singing songs such as "Five Little Ducks" or " Two little dicky birds, sat on a tree, one named Peter, other named Paul, fly away Peter, Fly away Paul, Come back Peter, Come back Paul (number operations), Can you forward, backward... sideways..?(geometry and spatial sense), The drum sound is rhythmic..da..da..da..can you repeat?(patterns), Can you dance slowly to the music? (measurement)

Children's progress :The teacher has to look for these indicators

Counting, making predictions or estimation, making rhythmic sounds or movement patterns, describing rates of movement(fast or slow), comparing sounds-same or different..

Mathematics in the outdoor area

Mathematics can be an integral and important part of most outdoor play experiences: counting each rung while climbing the slide's ladder, considering which tree is tallest, noting the striped pattern on a caterpillar, or crawling through a tunnel. Mathematics vocabulary easily becomes a part of outdoor conversations. for example, children talk about who will ride the tricycle first, second, or third; announce who pedals faster; explain how they wove around traffic cones, or tell the teacher about the length of a friends turn with a new toy.

As an early childhood teacher, one should play an important role in bridging children's informal understanding of mathematics with more formal school-based mathematics. The teacher has to design learning environment by purposively placing mathematics material in the interest areas for child-initiated explorations. The teacher has to plan logic games, create mathematical problem solving stories, include numerical and algebraic activities as part of daily routine.

2.3.6 : How and what mathematical concepts are to be taught to preschool children ?

The National Council Of Education And Research Training (NCERT) designed content standards in five areas for preschool children.

- ✚ Number and Operations
- ✚ Geometry and Spatial sense
- ✚ Measurement
- ✚ Patterns (algebra)
- ✚ Data analysis

The above components of mathematics are dealt in detail in the subsequent chapters.

Number and operations :number concepts are the most identifiable mathematical concept in preschools. At the preschool level, number and operations concepts involve nine different ideas.

- Counting :In order to count well, children must learn three things :number sequence, one to one correspondence, counting set of objects. Young preschoolers first learn to understand the words one, two, three four and to identify collection of objects that represent those numbers.
- One to one correspondence means that one number name is given or matched to one and only one object in a set is being counted.Finally children must realize that the last number named when all objects in a set have been counted is the number that tells how many. This is called cardinal number.
- Quantity (number sense) :Understanding how many are in a set is one of the first ideas the child gets.The child may show incorrectly his fingers when asked about his age which he develops easily with experience children often look at a group of objects or fingers extended on one hand and identify the quantity without even counting.This is called **subitizing**..With experience children however understand the quantity and learn to count the objects in a set or making sets.
- Comparison (more/ less/few/ tall/ short/ heavier/ light) Children often tell which set has more or less by visual discrimination. Children should be able to use words like one or more or two more.
- Order :ordinal numbers first, second, third and so forth indicate where someone is in the linear the position of items in a row of objects.
- Numerals : children need to investigate the use of numerals which is equally important as alphabet letters. It is important for the preschool children to see the numerals displayed though writing numerals is not important at this age.
- Combining operations (adding) children add with symbols initially and later learn to add or combine sets to find out how many in all.
- Separating operations :subtractions :It is a child having a set amount of objects and some objects being removed and how many are left over.
- Sharing operations (dividing) : children often share snacks and other things . when they begin to share in sets they understand the operation of dividing or forming groups.

- Set-making operations (multiplication) :They make equal sets when they give everyone two cookies or place stting in the dramatic play.

Role of teacher : Teachers should identify every day situations that involve numbers and intentionally introduce number concepts through daily routines, choice time activities , large and small group instructions.

Provide a variety of materials to help children develop understand ing of quantity.

Demonstrate how to count objects by keeping in a line and model strategies.

Model comparing the number of objects in two sets

Identify every day situations to use ordinal numbers.

Encourage children to tell stories as many as possible.

Geometry and spatial sense

Young children find geometry an exciting topic. In pre-school, there are four important geometry concepts, young children need to explore and level up.

Shape: Both two- and three-dimensional shapes are important to the understanding of geometry. Young children need to recognize shapes, build with them, illustrate them in their own way, describe shapes attributes, compare shapes and sort them by their characteristics.Finally, young children should be encouraged to predict what will happen when they build and create with shapes or put together two- or three-dimensional shapes.

What is the teacher’s role in developing and understanding of geometry?

Specific teaching strategies can be used to support young learners’ geometry.

Provide opportunities to all children to use the Block Area

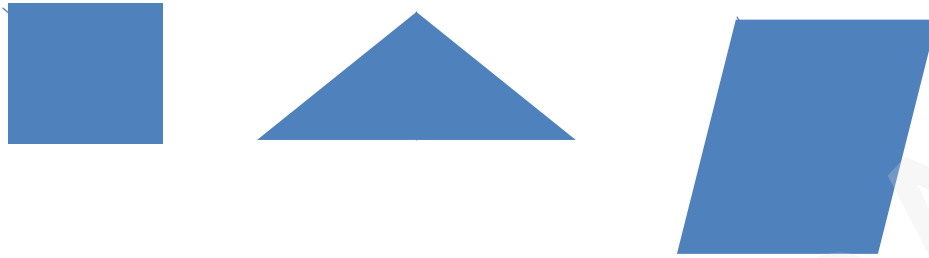
The Block Area is the perfect place for the children to explore the attributes of three-dimensional shapes. For example, when children stack blocks, they naturally investigate the surface to see if it is flat and can stack easily or if it is curved and cannot stack. All children should have access to this important area, so accommodations may have to be made to enable children with disabilities to use the area easily.

Label Shapes with correct names as the children use them

Use the geometrically correct names for shapes. This can be done by simply adding vocabulary to the child’s descriptions or manipulations. For example, when a child says, “I got a round one,” when describing a sphere, you can say, “Yes, it is round. It looks like a ball. I call it a sphere.”

Ask Children to predict and investigate what will happen when two shapes are combined

Introduce activities that require children to match sides or surfaces of two shapes. Asking children to make a tall smooth tower out of unit blocks encourages them to predict and then investigate how to accomplish that task.



Model and describe how to make two- and three-dimensional shapes

Create a particular shape using clays, paper or smoother flexible material and describe it as you work. For example, as you transform a clay ball into a cylinder, you can say, “I am rolling the clay to make the sides smooth. Now, I am patting it on the ends so that it will have flat circles on the top and bottom.”

Have children clean up by placing shapes on a shelf or in a box so they can easily fit

Opportunities for clean up abound in an early childhood classroom.

When shelves in the Block area are labelled as described in *The Creative Curriculum for Pre-school*, children can match the blocks with their two-dimensional representation. In the toys and games area, containers can be labelled so that children can place all the triangles together, all the circles together or all the squares together.

Use the word not to introduce non-examples of specific shapes

To fully understand the attributes of particular shapes, it is critical that young children know what shapes are not classified as a particular shape. Create many opportunities for children to sort shapes into two groups, those that are the shape and those that are not shapes.

Make class maps and have children use them to find particular objects

Give the children a map of the class room with identifiable landmarks and specific clues about the object’s location. Have the children search for the hidden object by using the map. For example, if the object is hidden under a box like waste basket, there could be a sketch of rectangular prism with an arrow indicating under.

As children work puzzles use words like turn, flip or slide to explain how the pieces might fit

Transformational language for young children includes the words – turn, flip and slide.

Begin with three dimensional shapes

Children need to hold and manipulate objects before they work with pictorial representations of the objects. Three-dimensional shapes should be used to introduce two-dimensional shapes. For example, making block prints in water, paint, or clay.

Provide a rich variety of shapes for investigation

Unit blocks are essential for teaching and learning geometry concepts. A variety of other three-dimensional shapes are important as well. Hemispheres, triangular prisms, triangular pyramids, rectangular prisms, square pyramids, and spheres provide many other contrasting experiences for children.

Provide activities that ask children to visualize and represent particular shapes

Show children photos, models or sketches of particular shapes or combination of shapes. Ask children to look and remember what they have seen; then hide the representation.

Measurement

Measurement is an important, practical mathematical concept that you often hear children discussing with their peers. Children begin to model measurement behaviors and frequently experiment with both standard and nonstandard tools. There are three measurement topics that should be explored by preschool children.

Measurement Attributes

They begin to recognize the attributes of length (how long or tall something is), capacity (how much something holds), weight (how heavy something is), area (how much space is covered), and time. Before children learn how to measure, they must first be able to describe and differentiate the attributes of an object by length, capacity, weight, and area.

Comparing and Ordering

Comparison is a fundamental concept that enables children fully to develop an understanding and use of measurement. They begin by comparing two objects by specific attributes: describing one object as taller or shorter than the other, holding more or holding less than the other, heavier or lighter than the other, or covering more or less space than the other. Next, children compare three or more objects or events and place them in order.

Measurement Behaviors and processes

The process of measuring is based on three fundamental concepts:

- **Conservation** – a set maintains the same quantity no matter how its parts are arranged or rearranged; an object maintains the same length if it is bent; an amount of liquid poured from one container into a differently sized container retains the same quantity

- **Transitivity** – if length A is less than length B, and length B is less than length C, then length A is less than length C
- **Unit**- the number and size of units is used consistently for the measurement of one object.

In preschool, experimentation with measurement behaviors is essential to mathematical understanding.

What is the Teacher’s role in developing an understanding of measurement?

Provide many standard measurement tools for children to use.

Standard measuring tools should be a part of the classroom environment. Rulers, yardsticks, meter sticks, measuring tapes, balance scales, centimeter grid paper, and marked measuring cups are tools that should be accessible to children in the classroom. Children should be encouraged to use them as they want for their measuring experiences.

Model measuring behaviors frequently

To help children develop an understanding of measurement, activities need to be modeled explicitly for children. Ex: Measuring table, floor etc.

Talk about what you are doing as you measure

Talk aloud as you model the measurement activity, to help children focus on the activity and the particular measurement strategy that is being used.

Encourage measurement problem –solving activities.

Take advantage of daily experiences to discuss measurement concepts.

Ex: - How much time is left for choice time?...How much longer until outdoor time?...when is snack time?...is it time for clean –up ”? They are also perfect opportunities to introduce a timer.

Use estimation vocabulary

Patterns (Algebra)

Algebraic concepts are key to a good basic understanding of mathematics. The recognition, creation, and extension of patterns and the analysis of change are important pre- algebraic concepts for pre school children. The study of patterns and change are exciting topics for young children and can be a strong motivation for discovery and creative thinking.

Patterns

Children begin to identify patterns in their environment at an early age. A consistent daily schedule, the phrases in a song or verse, or the repeated colors of the wall tiles are all patterns that can be easily recognized and described by young children. Extending those patterns in a consistent way is a skill that can be taught to young children and, with practice, transferred from one representation to another. Patterns in sequences of sounds and movement (e.g., stomp, clap, stomp, clap, clap; stomp, clap,

clap...), colors in a striped shirt (e.g., blue, red; blue, red; blue, red...) and shapes and positions in a block wall (e.g., block up, down; up, down; up, down...)



Are examples of repeating patterns because each unit is repeated in a consistent way?

What is the teacher’s role in developing an understanding of patterns?

Many young children naturally search for patterns, but their discoveries need to be labeled and extended. The teacher’s role is to challenge children to identify patterns in many settings, represent those patterns auditorially and with objects, and extend those patterns in consistent ways. In addition, the teacher should provide many opportunities for children to create their own patterns with objects, sounds, or words and purposefully teach children different representations for the patterns they identify.

Identify different patterns in daily routines.

Encourage pattern “talk” and identification.

E.g., “Let’s walk like an elephant, swaying back and forth and moving our trunks: right, left; right, left; left. Oh, it’s a pattern!”

Point to numerals as you count out loud.

Rote counting is a familiar pattern. The number sequence of 1, 2,3,4,5,6,7,8, and 9 repeats as children count higher and higher.

Begin with color patterns and progress to shape and size patterns.

Color patterns are the easiest patterns for young children to identify. Help children create patterns by isolating one attribute at a time. For example, encourage children to use objects that are identical except for color to help them create color patterns. Next, have them use objects that are the same color but different shapes. Continue the sequence with same- colored, same- shaped , and differently sized objects.

Describe positional patterns.

They provide a good opportunity to take spatial terms that are critical to geometric understanding and connect them to algebraic ideas. Block or object patterns can be described using words like up, down, right, left, high, low, crooked, or straight. The encourage the use of positional patterns; provide children with identical blocks or objects. This will help them to focus on the attribute of position rather than color, shape, or size.

Focus on the unit that is to be repeated in a pattern

To help children focus on the pattern unit, children can “become” part of the unit. Ex: one child, role-poly roly-poly right , right , right another child, Roly poly..... up, up, up.... Roly, poly.... Down, down, down.

Use patterned stories and verses.

Many stories and verses repeat events or phrases in a patterned, consistent manner. These patterns can be illustrated and extended by writing additional parts for the verses or stories.

Create pattern and change books

Encourage children to represent patterns they have discovered or created by making books with illustrations of the patterns.

Provide opportunities to observe change.

Use a variety of representations for patterns.

Extend pattern units for at least five units before a pattern is established.

Data analysis

Many pre schools classrooms contains teacher – made graphs and pictures of data collected by children. In preschools there are three important ideas that involve concepts of data analysis.

Shorting and classifying

Using the attributing of objects to sort and classify is an important skill for young children in many content areas.

Representing Data

In preschool classrooms, data is normally represented by using concrete objects, pictures, and graphs. The goal of graphing with young children is to provide a way of showing data about a child or his surroundings so it can be seen and understood. If displayed and labeled properly, children can make comparisons and describe what they see.

Describing data

Vocabulary like more, fewer, the same number as, larger than smaller than, and not can be used to describe data displayed on a graph or picture. These terms help connect the topic of number with data analysis.

What is the Teacher’s Role in developing an understanding of data analysis?

A rich variety of experiences, particularly those involving sorting and classifying, help children to develop the concepts and skills that underlie data analysis. Experiences that help children pose

questions, collect data, organize data, represent data, and describe data are all very important aspects of this topic.

Use classroom routines to represent data.

Encourage children to organize objects using their own rules.

Purposely describe collections in more than one way.

Ask children to line up in classification groups.

Use two groups to organize data.

Use paper of the same size to create bar graphs.

Demonstrate classification and ask children to “guess” the describing words.

Use symbols to represent data.

There is a feeling among large sections of adults including teachers that mathematics is a serious subject and should be taught with all seriousness and there is no place of any light hearted activity in teaching and learning process. But, for young learners at the initial stage of schooling such a serious approach has invariably causes loss of interest in mathematics, developing a fear complex towards the subject and ultimately resulted in early drop out from school .Children can learn most of the basic concepts of mathematics being engaged in activities that give them pleasure. Every child loves to play games with other children and these games that may be the perfect medium for learning several mathematical concepts.You can take any familiar game children love to play and with slight modification you can integrate some mathematics concepts in it so that the children can acquire those concepts while enjoying the game. In addition to these efforts, you can devise interesting activities specifically for the purpose.of introducing math .

2.3.7 :End Summary

Most children enter preschool knowing a lot about math. In a safe and supportive classroom they will feel comfortable taking risks and engaging in self-directed problem solving. Weaving math into all areas of the curriculum will heighten children’s play experiences and allow all learners to experience success. Children will soon see themselves as mathematicians who apply their skills in a number of ways. Their growing math skills, confidence, and interests will serve them well in school and life. To develop mathematical thinking in children , the role of teacher is central in providing intentional math concepts in the activities thus encouraging children to use math vocabulary to understand math concepts in a comprehensive way. The teacher has to make the classroom math rich to encourage thinking in children. Thus the teacher plays a major role in making informal experiences into meaningful experiences for children

2.3.8 :Unit End Questions : Q1) What are spatial skills? How can a preschool teacher promote spatial concepts in children ?

Q2) what is the role of teacher in promoting number concepts in preschool children?

Q3) What are the mathematical concepts involved dramatization?

Q4) What are the measurement skills attained in sand play?

Q5)As a preschool teacher, how do you introduce shapes concept to children ?

Q6) What are the five different components of teaching mathematics.Explain briefly with suitable examples.

Q7) What are the different steps involved in learning counting ?

UNIT 3 : Matching and Classification

3.0 Introduction

In the earlier units we have learnt how young children show intuitive discrimination up to 3 (subitization) and approximate discrimination of large numbers. We also discussed how everyday experiences in child's life can lead to the foundation for development of mathematical concepts in early years.

From the above learning's we have understood that math is every where around the children and it is not just learning of numbers. The kind of every day math related experiences- say it through hands-on experiences, rhymes, songs, stories; create a solid foundation for children to understand many mathematical concepts. Children who have the opportunities to learn math related concepts in early childhood often do better in school and are more skilled at using math as an adult.

3.1 Objectives

In this unit you will be able to learn

- how to lay solid foundation for preschool children in learning of mathematics
- how important developing cognitive skills at preschool – matching and classification for later learning of math in higher grades.
- to plan different activities for developing the skill of matching and classification in young children.

3.2 Early Childhood- Foundation for Mathematical concepts:

From the discussions from unit 1 and 2 you may have developed a lot of ideas about how to facilitate children's learning of mathematics concepts at the early stage of their development and also at the early stage of schooling. Young children construct mathematical ideas on their experiences with their interactions with the environment, adults and other children and also with their daily observations

3.2.1 Ways of Learning Mathematics

There is no single and definite way of learning mathematics even at the earliest stage of learning. From the previous discussions in this unit, you might have formed some ideas as to the nature of learning mathematics at the early days of schooling. In addition to those, here are some points regarding the characteristics of nature of mathematics learning as summarised by White Bread (Anghileri, 1995):

- Mathematics starts from 'home learning' established in the child before he/she comes to school.
- Mathematics is based on understanding.
- Mathematics puts great emphasis on the child's own methods of calculating and solving problems and rejects the previous practice of heavy emphasis on standard written algorithms
- Mathematics is regarded as a powerful tool for interpreting the world and therefore should be rooted in real experience across the whole curriculum.
- Mathematics is brought out of the child's everyday situations.
- Mathematics with reason is rooted in action – learning through doing.

- Mathematics with reason puts less emphasis on representing numbers on paper as ‘sums’ and more emphasis on developing mental images in the child.
- The main tool for child and teacher to employ in the mastery of mathematical concepts is language, not pencil and paper exercises from textbooks. The child is encouraged to talk about what he/she is doing.
- Errors are accepted as essential part of the mathematics learning process. The child, who is free from the fear of criticism, will more readily experiment

From this, we understand some basic ways related to the learning of mathematics during the early stages of school learning.

The following are the ways of learning which as a teacher you can facilitate in the classroom situations.

a) Manipulation of objects: As you have observed, it is through the manipulation of concrete objects that the children acquire the mathematical skills at the early stage. Acquisition of any mathematical skill at the early stage like comparison, categorization, counting, fundamental four operations, cannot be possible without manipulation of concrete objects. Provision of a variety of objects both familiar and novel, should be made available to children in the classroom for their free handling, so that it would be easier for you to facilitate their learning of desired mathematical concepts. Lets recollect the Piagetian theory of Cognitive Development in children. It clearly suggests that children move through four different stages of mental development. His theory focuses not only on understanding how children acquire knowledge, but also on understanding the nature of intelligence. The stages are:

- Sensorimotor stage: birth to 2 years
- Preoperational stage: ages 2 to 7
- Concrete operational stage: ages 7 to 11
- Formal operational stage: ages 12 and up

Piaget believed that children take an active role in the learning process, acting much like little scientists as they perform experiments, make observations, and learn about the world. As kids interact with the world around them, they continually add new knowledge, build upon existing knowledge, and adapt previously held ideas to accommodate new information. The first two stages fall under Early Childhood Education period, which clearly state that children acquire knowledge through sensory experiences and manipulating objects. A child's entire experience at the earliest period of this stage occurs through senses, and motor responses. While they get better with language and thinking, they still tend to think about things in very concrete terms. Hence manipulative activities are essential at this stage of development.

b) Reasons for Using Manipulative

As part of the problem-solving process, children use concrete materials as aids in modelling concepts, formulating relationships, and improving understanding.

Manipulatives:

- make children's mathematical thinking visible, so that everyone can see it, talk about it, and learn it;
- provide a context for developing mathematical concepts;
- help children explore, think about, and talk about mathematics;
- help children construct meaning and see patterns and relationships;
- allow children more easily to test, revise, and confirm their reasoning;
- help children make connections between concepts and symbols;
- help children talk about the math, with the result that teachers have a basis for assessing children's understanding and can make programming decisions based on their observations.

When using manipulatives in a Preschool classroom, the teacher should:

- select materials that are appropriate for the developmental level of children.
- introduce manipulatives explicitly and refer to them by name
- establish rules for distributing/replacing manipulatives as well as routines to ensure that children are not interfering with others and are using the materials in respectful manner
- provide time for children to explore the manipulative that will be used
- ask children what they discovered about the manipulative
- start with just a few manipulatives and do several activities that help children see the wide range of uses each manipulative has
- think aloud when using manipulatives, so that children will learn to verbalize what they are doing
- question and prompt children as they are working with manipulatives to assess understanding, encourage talk, extend thinking, or consolidate learning
- ensure that manipulatives are always available for children to use when solving problems and exploring concepts, and encourage their use

The foundation of mathematics is set during the early years. In these years, teachers use manipulatives to help students understand mathematical concepts in a way that accommodates students' need to explore with their hands and body in order to learn.

Hence as preschool teacher you need to focus on planning lots of hands on activities, working with concrete material using the locally available and culturally and developmentally appropriate. Exp...Flowers / fruits /vegetables/ pebbles / sticks / leaves / clay / seeds / toys / household utensils etc.,

- c) **Placing tasks in meaningful contexts:** In real situations, where the mathematics serves real purposes, young children quickly and easily develop their own informal and largely effective methods. Evidence from research about the ways children learn seem to suggest that what we need to do is to start with real problems, and work from them to abstract representations. There are abundant opportunities in the everyday activities of young children to get themselves

involved in real mathematics. Playing games, sharing things, groupings in the class for performing different activities, are just a few such examples.

- d) Representation in multiple ways:** Another important element which is required to help children move towards abstract thinking in mathematics involves helping them to develop their representational abilities. Many mathematics educators now believe that it is important that children express their mathematical thinking in language, through talk, before they begin to represent it on paper and before they use mathematical symbols.

In the coming sub units we will discuss different activities based on the above principles.

3.2.2 Making Mathematics Learning Pleasurable

There is a feeling among large sections of adults including teachers that mathematics is a serious subject and should be taught with all seriousness and there is no place of any light hearted activity in teaching and learning process. Children can learn most of the basic concepts of mathematics being engaged in activities that give them pleasure. Every child loves to play games with other children and these games that may be the perfect medium for learning several mathematical concepts. You can take any familiar game children love to play and with slight modification you can integrate some mathematical concepts in it so that the children can acquire those concepts while enjoying the game. In addition to these efforts, you can devise interesting activities specifically for the purpose. Here is an example:

Game: Matching skill:

Divide children into 2 groups. Make 2 sets of pictures of different flowers which children can identify. Pin up the cards to their dress. Ask them to run around a circle while you make sound with dappli. Stop the sound and tell them “children look at the picture and go find out who is having the same picture and stand together.” Isn’t it fun for children to play and they never feel that it’s a lesson on matching.

3.3 Mathematical concepts in Preschool (Emergent and Early Mathematics)

As discussed earlier play and activity based approach , with balance of free play and guided activities; group and individual activities; active and quiet activities; outdoor and indoor activities with and without materials the preschool programme has to be planned.

After understanding how to develop mathematical skills in preschool, now let’s think a while, what should be included in Preschool math’s keeping in view the children’s stage of development, their exposure and experiences.

The following are the essential components of Preschool mathematics, which children need to acquire before they start learning the numbers and the fundamental operations.

A) Pre -mathematical Concepts

- Relative locations/positions/space
- Quantitative dimensions (pre number)

B) Number Concepts

- Classification/set
- One to one correspondence
- Equivalence
- Development of number concept through rote and rational counting
- Concepts related to Number properties and number operations
- Estimation

C) Cognitive skills

- Reasoning
- Problem solving
- Seriation
- Sequential thinking
- Pattern making

D) Approach to learning

- Interest in learning/ playing with numbers
- Perseverance on task
- Working memory
- Emotional regulation

These are the fundamental skills which need to be mastered in preschools before working with real numbers.

As adults we sometimes overlook the importance of these fundamental skills in a young child. When some people talk of mathematics learning they routinely speak of number sense or numeration, and more specifically counting or even the higher order operations of addition, subtraction, multiplication and division.

However, in the young child there are many beginning process skills which need to be understood before they can engage with number. These include identifying and describing attributes, matching, sorting, comparing and ordering.

A child must be able to classify objects into groups before they can take part in significant number learning. Children need to learn how to sort and classify before they are able to move on to activities that involves numbers because they need to know what they are counting before they are able to actually count them.

As with all skills there is also a developmental sequence of understanding matching and classification..

- Identifying and describing attributes- children need to be able to notice the attributes of a particular object. They need to be able to notice the details and to be observant. Identify objects which are similar / different.

- Matching- here children are focusing on the sameness of an attribute. Match objects on the basis of a given perceptual (visible) attribute – say colour, shape, size, texture. And match objects on the basis of function- say things we eat /things we do not eat .They need to develop the language which helps them explain their thinking. Children need to be able to express why they are matching something. Matching nurtures children’s reasoning skills.
- Sorting- the ability to sort using two or more attributes (characteristics) represents higher order thinking. Children need to look at the characteristics of different items and find characteristics that are the same. Young children usually begin sorting by color before sorting by other attributes
- Classification is a fundamental pre-number learning concept that children learn about the world around them. Classifying and sorting can be done with or without using numbers, such as separating children or objects into distinct groups, such as the colour of their dress, or their lunch bags. Children need to learn how to sort and classify before they are able to move on to work that involves numbers, because they need to know what they are counting, before they are able to actually count them (Reys, et al., 2012).

Now we understood that while planning the math programme for preschool children one has to keep in mind the sequence of skills those children are to exposed/experienced before working with classification activities. This helps children to learn the math skills with confidence which leads to pleasurable learning experiences in preschool.

3.4 Identifying and describing attributes- Same- Different:

Mathematics is a fascinating subject. The tool we use in everyday life to sort, count, measure and analyse, are all element of Mathematics. When it comes to teaching Maths to the Preschoolers, the concept extends from numbers to patterns, shapes and some basic pre-math concepts. Learning Maths is important for young children. It gives them the confidence to explore and develop an understanding of the world around them.

During their fourth year, children begin to compare things in terms of similarities and differences. This is an important analytical skill that leads to classifying and other reasoning abilities.

Why is this important? ‘Same’ and ‘different’ are concepts used to describe similarities and differences between objects. They are frequently used across the curriculum and are used in the context of numeracy and science in particular.

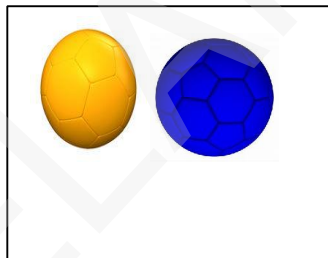
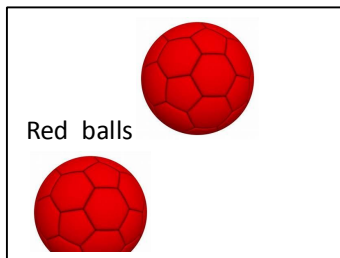
Hence children are to be introduced to the concept of “Same and Different” through some interactive activities like match the same fruit or colour the different one, to expand their understanding of pre-math concepts. These activities help them to enhance their logical, visual, fine motor and gross motor skills.

3.4.1 Finding out similarities

”We have already discussed in various papers that young children learn through interactions with the environment, people and age appropriate materials. Now let’s look into the way children identify similarities and differences.

Visual discrimination- Through visual perception child identifies the differences and the similarities in a given set of objects. Take a tray , keep one red and one blue ball in it and hold one red ball in your hand. Before you ask the child to match, the child needs to know how to determine which ball is “the same”. Show the red ball in your hand and ask children to find one that is the same in the tray. Then keep 2 red balls in one tray and a blue and yellow ball in another tray .Point to the balls and ask the child to identify the object .At this stage you need not expect the child to say the colour of the balls.Ask the child to show which 2 balls look similar /same. Once the child points out to the 2 red balls, then ask children “how can you say that these two are same” Questioning helps children to think, and answer logically using the right vocabulary.

1) Same (colour)



Yellow ball

Blue ball

Give many experiences with objects / things that are available in their near surroundings.Exp

2) Same(pattern) –Identify which pair of socks looks same



1) Same (size)

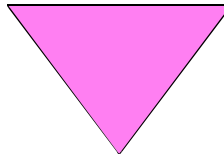


Encourage children to identify the picture and ask to show apples of same size .

2) Same (shape)



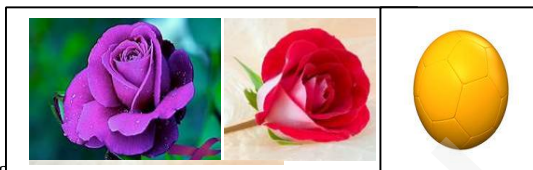
(Picture A)



(Picture B)

Encourage children to identify the shape in pic. A and ask to show same shape in the pic. B

5) Same nature



In the above example though the colour of rose flowers is different they are same because of the same attribute ie flower .This kind of examples have to be given for older children once they master the skill of classification.

6) Same (texture)

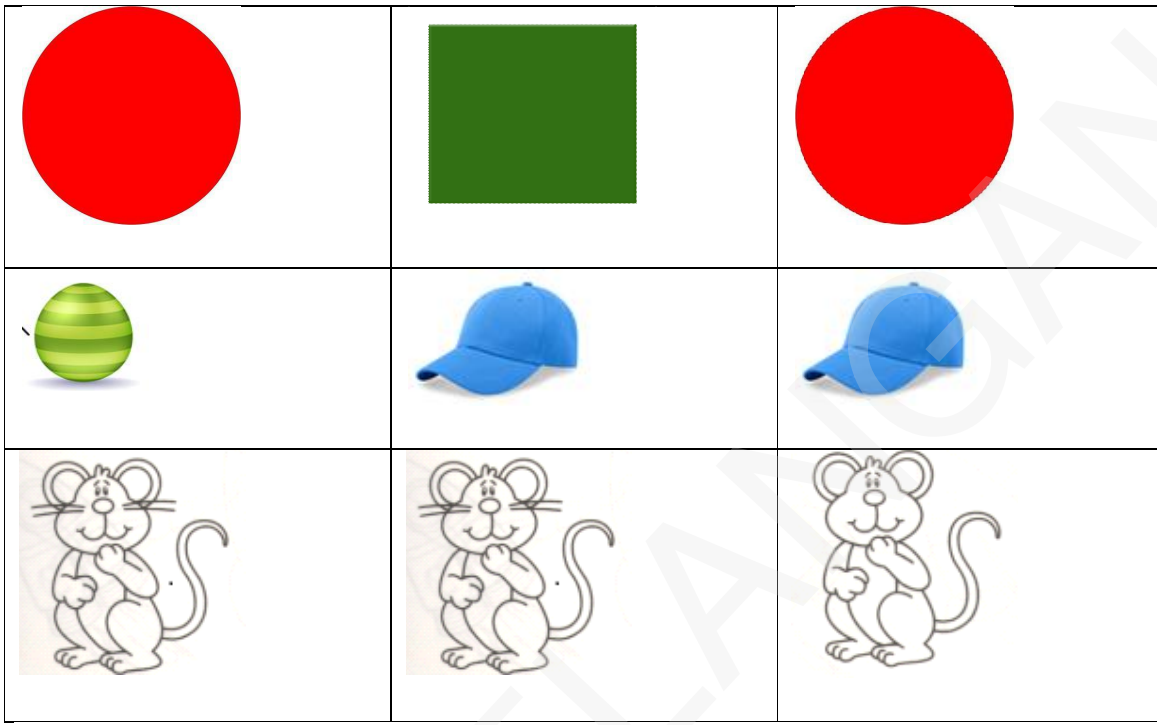
Feely bag: Collect same textured items say smooth silky clothe pieces and other clothe pieces which has rough texture. Call one child and ask to touch the silky cloth with hands and feel it. Then give the feely bag in which different textured cloth pieces are kept. Instruct the child to keep hand inside the feely bag and take out the same textured cloth piece which he/she was given to feel initially.

This activity can be given to 5 to 6 year old children.

Remember: You may plan activities for development of this skill – looking for similarities based on different attributes like- colour, size , shape , texture , use etc. Remember first provide experiences using concrete materials. Once the children understood the concept then provide worksheet / read stories / plan games for reinforcement of the skill.

7) Sample worksheet to find the similarities.

Circle the pictures that are same/alike

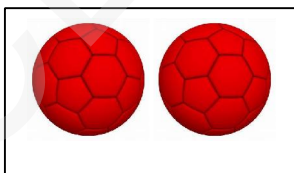


3.4.2 Finding out differences:

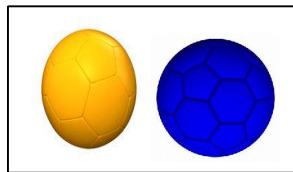
Identification of 'different' item based on colour, size, shape, texture.

Visual discrimination- Through visual perception child identifies the differences and the similarities in a given set of objects. Show 2 red balls to children, ask the child to identify the object. At this stage you need not expect the child to say the colour of the balls, then ask the child to show in which set 2 balls look different. Once the child points out to the yellow and blue ball set, then ask children "how can you say that these two are different" Questioning helps children to think, and answer logically using the right vocabulary.

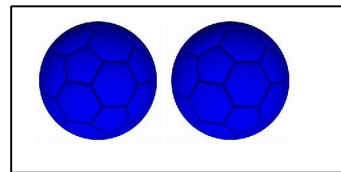
1 Difference (colour)



Red balls



Yellow Blue



Blue balls

Give many experiences with objects / things that are available in their near surroundings. Exp

2) Different (pattern)



Ask children to identify a pair of socks which has different design and say it verbally.

3) Different (size)

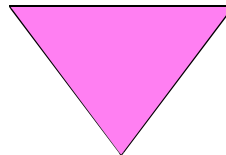


Encourage children to identify the picture and ask to show the one which is different from other two apples and encourage children to say it verbally (exp. this apple is different because it is big in size)

4) Different (shape)



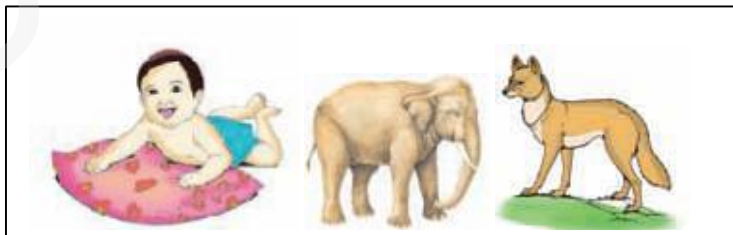
(Picture A)



(Picture B)

Encourage children to identify the shape in pic.A .Then ask to point out the shape in pic. B which is different from pic. A

5) Different



a)

In the above example (5a) ask children to identify the pictures .Then ask which is picture is different and why? Encourage children to explain their point of view.



b)

In the above example (5 b) though all pictures are of leaves .One of the leaf is different from other two. Encourage children to find out the difference and say it orally. .This kind of examples has to be given for older children once they master the skill of classification.

6 Different (texture)

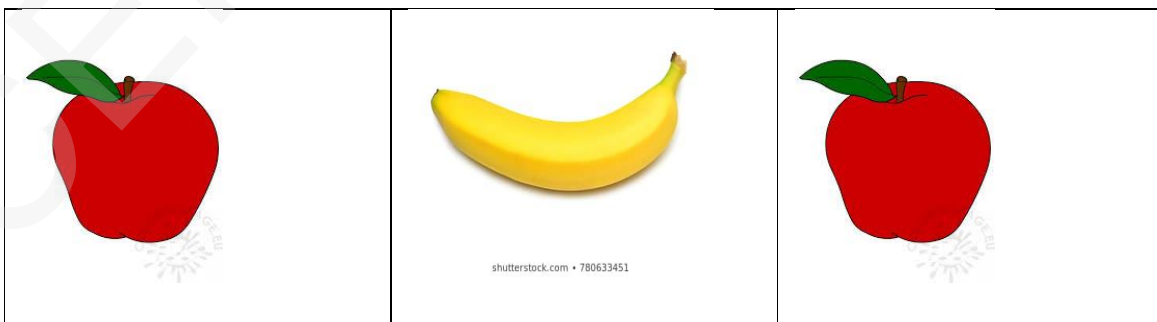
Feely bag: Collect 5 same textured items say smooth silky clothe pieces and 1 woollen / jute piece which has rough texture. Call one child and ask to put his/her hand in the feely bag. Instruct the child to keep the hand inside the feely bag, touch and feel the texture of all the cloth pieces and take out the different textured cloth piece .And then encourage child to explain the basis of identifying the different textured cloth piece. This activity can be given to 5 to 6 year old children.

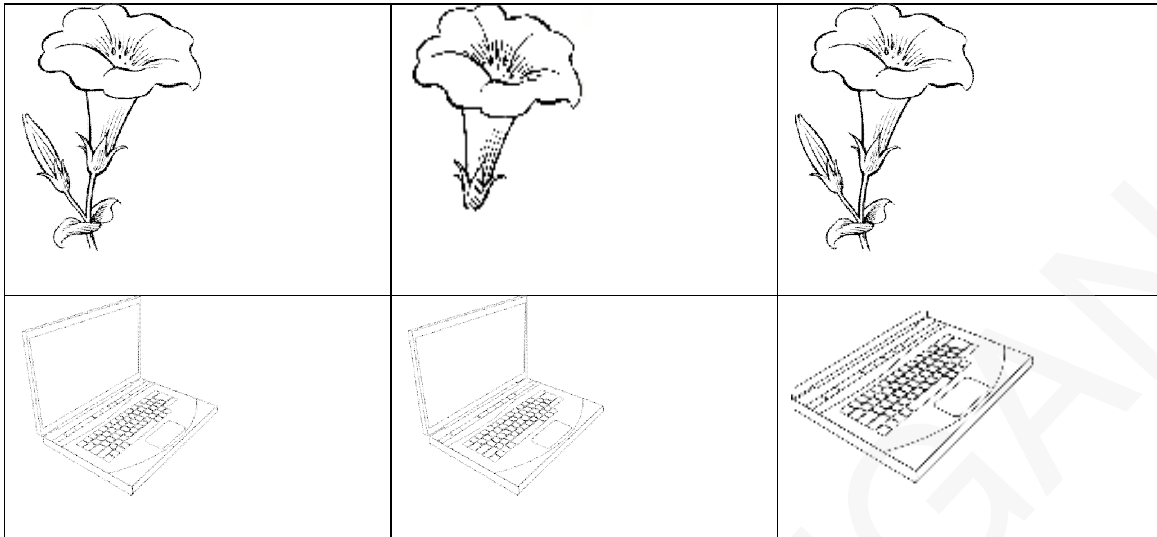
Remember to ask always “how can you say that these two are different” Questioning helps children to think, and answer logically using the right vocabulary.

Once the child understands and equips with skill of identifying same and different based on different attributes like colour, size, shape, texture give worksheets.

5) **Worksheets** - One example is given for you.

Draw X on the picture that is different





How to make understand children ‘same’ and ‘different’ concept attainment in children?

As teacher what you have to do is...

- Let the children do the activity, ‘Odd man out’. Prepare strips of three to four pictures with one different and three identical. Let them spot the one that is different.
- Make two identical pictures with only one prominent difference between them. Let the children spot the difference.
- Make ‘touch cards’ with different kinds of cloth pieces, for example, satin, jute(J), cotton(c), wool(W), etc. Let children match cards according to texture.
- Find pictures of objects in catalogues/magazines that are exactly the ‘same’ and pictures of things that are ‘different’, e.g. two identical shoes and one cup and one pencil .Place the two pairs of pictures on the table. Ask the child to point to the pictures that are the ‘same’ or those which are ‘different’.
- Real toys/objects can be used if you have got matching items (e.g. two identical balls vs. one bat; one car or two spoons vs. one plate and one cup).
- Introduce items that are different but only slightly (e.g. a red pencil and a blue pencil, two different-size spoons).Encourage the child to tell you whether items are the same or different. Reduce the use of signs and gestures so the child is relying solely on verbal understanding.
- Use a corresponding gesture or action for ‘same’/‘different’ to help understanding. The child may find it easier to distinguish between ‘same’ vs. ‘not the same’ to begin with, rather than

‘different’. Start with objects from different categories to begin with for the two different items (e.g. pen vs. elephant).

- Start by introducing the concept ‘same’ to begin with (e.g. matching the colour crayons that are the ‘same’; talking about two items of food that are the ‘same’, such as two similar biscuits). Once the child is clear with “same” then introduce “different” through concrete materials.
- **Odd-man-out**

Make strips with pictures of two objects of one kind for eg. Vegetables (brinjal, tomato). Have a third one which is different for eg. a fruit (banana). Encourage the children to find out which is different.

3.5 Matching

Once children are able to identify similarities and differences they are ready for matching objects based on various attributes.

Matching is an important early childhood math skill that helps in classification of objects. Matching is identification of same or similar objects based on their common properties. And this skill keeps recurring through out schooling life in various forms. For example, matching skills are used to identify congruent or similar triangles. In algebra, matching can be thought of as a one-to-one correspondence function between two sets. Important early matching skills that a young child needs to develop are:

1. Matching by Colour
2. Matching by Size
3. Matching by Shape
4. Matching by Texture

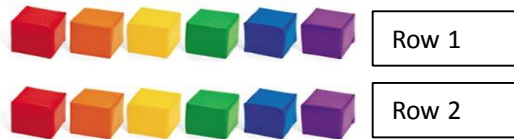
Matching- here children are focusing on the sameness of an attribute. Match objects on the basis of a given perceptual (visible) attribute – say colour, shape, size, texture. And match objects on the basis of function- say things we eat /things we do not eat .They need to develop the language which helps them explain their thinking. Children need to be able to express why they are matching something. Matching nurtures children’s reasoning skills. Now we know that it is not just match same looking objects/things, but it is much beyond simple matching.

Matching mainly involves one-to-one correspondence. Matching forms the basis for our number system .When a child can create “the same” it then possible to match two sets. This becomes a prerequisite skill for the more difficult tasks of conservation.

Activities for developing the Matching skill in young children.

3.5.1 Matching by colour (blocks)

Ask the children to match blocks of the same colour.



Stage 1 (younger kids)

- Arrange the colour blocks as shown above. Give children a basket / tub with different coloured blocks .Instruct them as pointing to the row 1 “children what are these? Does all the blocks are in same colour? Can you match each block with the same colour block? Let’s start Teacher will pick one blue block , holds up and asks children “show me the same colour block”. Once the children show the blue coloured block teacher will match it with the blue block in row 1. Encourage children to do it on their own. While children doing the activity you should ask them on what basis they are matching the blocks.
- Children may be encouraged to match each other’s clothes on the basis of colour.
- Give a red coloured thread / block / piece of cloth and ask children to find objects of the same colour in the environment.
- Make 2 sets of colour dominoes. Let children sit in a circle along with the teacher. Distribute one domino to each child keeping one set of dominos in the centre of the circle .Let each child bring his/ her domino to the centre and find the matching one. And children should be encouraged to tell the basis of matching.
- Make visual discrimination cards. Each card can have a given shape on the top and a simple scene below in which the same shape is used as a part of the picture. Let the children find the shape in the picture

Stage 2 (older kids)

- Arrange the colour blocks – red, blue, yellow, green in a row. Give children a basket / tub with red, blue, yellow, green blocks. Points to each block and ask them to identify the colour .Then instruct them as pointing to the row 1 “children what are these? Does all the blocks are in same

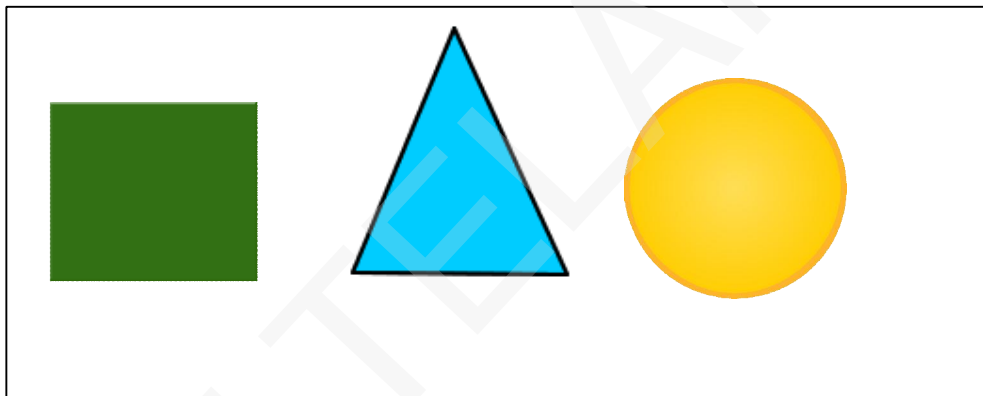
colour? Can you match each block with the same colour block? Encourage children to do it on their own. While children doing the activity ask them on what basis they are matching the blocks and explain using the names of the colours and words like same , alike which they have learnt in the earlier activities conducted for identification of same / similar .

You can use the locally available material like green leaves , different coloured flowers , and different coloured cloth pieces (can be procured from the local tailor),different colored toys that are there in the classroom etc.

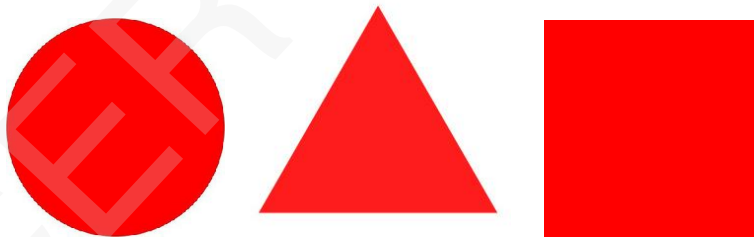
3.5.2 Matching by shapes.

Conduct the activity as discussed above, focusing on shape instead colour. Children are expected to match the shape with visual perception. Very young children not expected to say the name of the shape. Children of 4+ and above should be encouraged to identify the shapes and tell the names of the shapes.

Cut out strips of card board. Different strips should have different shapes in different colour. Make some cut outs of the shapes all in same colour. Let each child match the cut outs with the shapes in the strips.



(All shapes are in different colours)

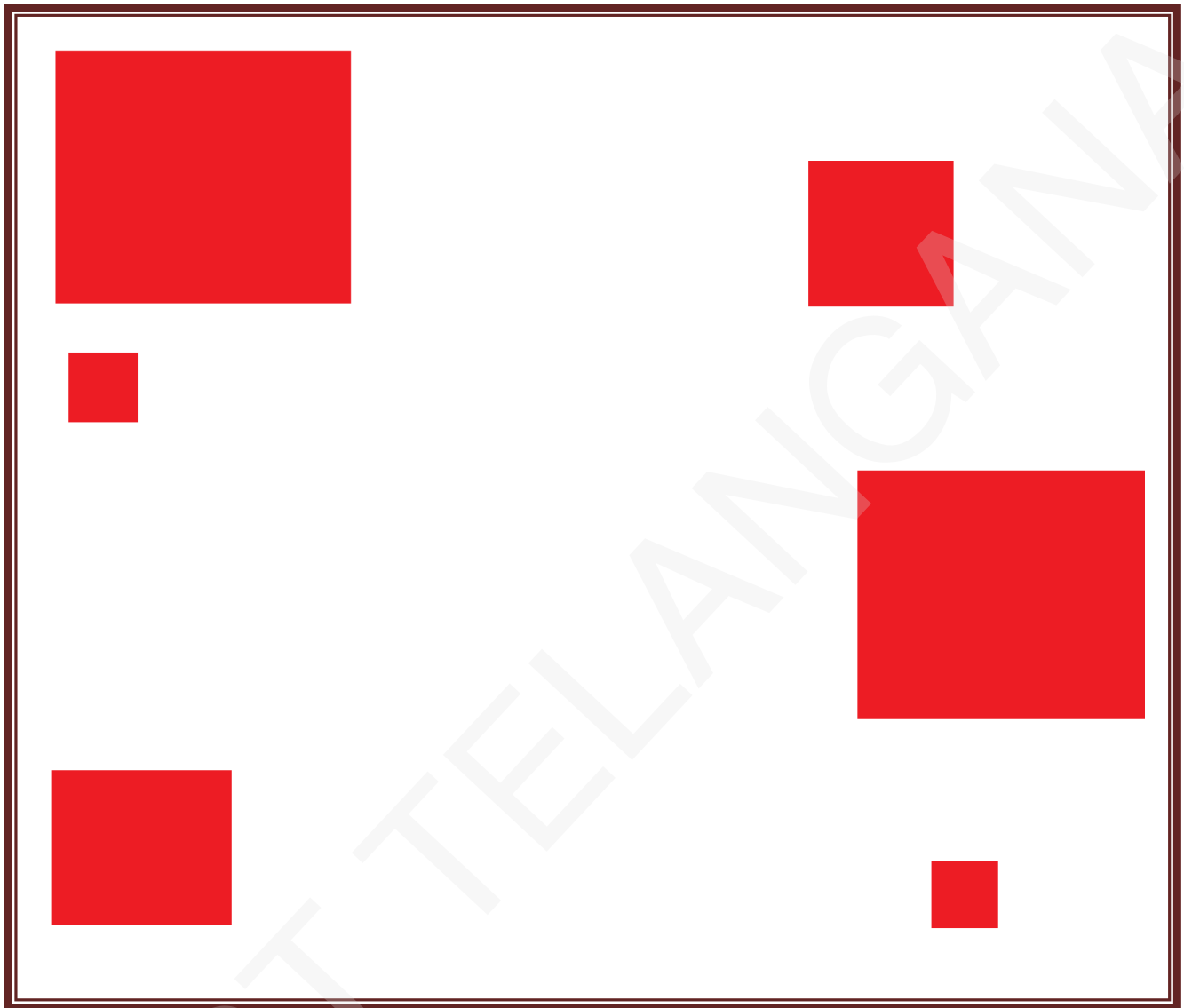


(all shapes are in red colour)

In this activity the focus is on the shape and not on the colour. Encourage children to match the red colour shape cutouts with corresponding shapes. Ask children to explain verbally on what basis they have matched and identify the shapes as circle , triangle , square.

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5.5.3 Matching by size.



Collect 2 sets of 3 different sized cut outs as shown in the picture. Encourage children to match the cutouts based on the size. Before doing this activity give experience with 3 dimensional material for example balls of different sizes.

3.5.4) Matching by Texture

Collect cloth pieces of 2 sets of different textures – jute, silk, and woolen, silk or prepare touch tablets using different grades of sand paper or procure touch tablets. Help children to match according to the texture and ask them to explain on what basis they have matched.

Locally available fruits and vegetables can also be used for this activity. Exp- bitter gourd and ridge gourd (rough) Cucumber and tomato (soft) etc., .

3.5.5 Matching real objects with the pictures

Children need to identify the pictures and match with its real objects. This helps in forming of mental images in children.

For Exp: Get the pictures of common fruits like banana, papaya, Guava , Orange , apple and the real fruits. Encourage children to identify the picture cards and match with the real fruits.



Real Fruits



Pictures of fruits

3.5.6 Matching on the basis of function:

Once children are able to match things based on different attributes, then you can ask the child to match things based on the use of the material .For Exp: matching on the basis of what we wear and things we do not wear.

And also encourage children to match things by its use by circling the pictures

Exp- 1 Things we eat and do not eat – apple , orange ,

2 Things use for travel - car , cycle

Where ever possible provide real things for children to match and encourage them to talk about the basis on which they have matched the objects / pictures.

Importance of matching skill in later mathematical learning.

- Benefits of learning matching skills in pre-school will support children to match letters and sounds at later stage.
- Being able to match items and explain why they go together is important for cognitive skills and ability.
- By matching objects to pictures children are practicing visual discrimination, becoming familiar with one dimensional print and learning to connect real objects to print. All of which are important pre-reading skills.
- Matching games improve language, concentration and memory.
- Matching skills are involved in visual discrimination. Children use matching skills to tell whether two words or letters are the same or different. Learning to match shapes and patterns helps children as they learn to recognize letters and then words.

3.6 Grouping /Classification: First Steps towards Logical-Mathematical Thinking

Classification is an important science and math skill. Classifying refers to organizing things into groups based on similar characteristics. There's a branch of science that deals with classifying organisms into groups. We call that taxonomy.

Simple sorting and classification are fundamental concepts that help children to organize their thinking about the real world (Reys, 1995). For example, with the development of simple sorting and classifying, children begin to differentiate between plants and animals, day and night, circle and square, and one and ten. Children begin to apply logical thinking to objects, events and mathematical concepts they encounter

Think about how classification is used in everyday life. Do you ever make lists? Do you make several lists – a grocery list, another store shopping list, a to do list inside your house, a to do list for things outside, etc.? That's a form of classifying. What about organizing things in your home? You group similar things together to make it easier to find. That's using classifying skills, too.

Why is classifying and sorting important in the Early Years?

Historically, with a perspective of learning as sequential relating closely to age, this resulted in the construction of developmentally appropriate practice (DAP). However, more contemporary perspectives such as constructivism, views children as innately capable and maintains that learning leads development as opposed to development ruling the learning (Arthur *et al*, 2014). As a result, children should be exposed to mathematics and provided with rich, open-ended numeracy experiences from a young age to support their development, rather than waiting until they are deemed developmentally ready. Classifying and sorting, as well as all mathematics experiences, are most effective when

incorporated authentically into children's everyday life. It is important for children to see mathematics in the real world to understand its place and uses (Harris, 2013).

More specifically, early mathematical concepts such as classifying, sorting, comparing and the problem solving that is involved to do these are the foundation for later, more complex mathematical thinking (Damon, (n.d.). Classifying and sorting is important for developing numerical concepts and the ability to group numbers and sets, important for completing complex sums in the upper primary years. By classifying and sorting, logical thinking is applied to everyday objects, which is important for later mathematics as well as all decision making (Renee, 2011). Significantly, studies have shown that competence in numeracy stemming from the early years increases academic performance in all subject areas in subsequent schooling (Daraganova & Ainley, 2011).

What does classifying and sorting involve?

Classifying and sorting involves finding things that are the same, or alike, and grouping them by specific traits or attributes. For example, a bunch of animals can be grouped based on the type of animal exp- domestic or wild. You can have young children classify anything, including blocks, leaves, plates, or toy cars. Once they have classified items, children can compare items further to learn more specific similarities and differences between items, both within and between matched groups (Raney, 2015). And according to Harris, (2013), classifying and sorting involves three steps:

1. Children decide which characteristic to sort by
2. Children physically sort the objects
3. Children are able to provide and describe their rationale for their classification

Is classification and sorting same?

Sorting versus Classifying

Sorting and classifying are terms that are often used synonymously when dealing with pre-number tasks associated with grouping objects or attributes. Some theorists including Piaget (Lavitelli, 1970) make a distinction between simple sorting and classifying.

Like all other math skills, sorting skills are developed in a **sequential manner**. Children begin by comparing objects and matching like items. Then they learn to categorize, which is sorting. They typically start with sorting by colour, then moving on to sort by type.

3.6.1 Sorting

Organizing like things together into logical groupings is what sorting is all about. Simple sorting is viewed as a beginning type of grouping task in which the way objects are to be sorted is shown or told to the child, "give me all blue cars ". Children are given or told the grouping pattern for the objects. Exp: Keep all different coloured toy cars in a tray. Instruct a child to pick up blue car and keep it in a basket. In the process the child is looking for the "blue "colour car from among the group that is the

child is sorting out the “blue car” from the group. This is the beginning for grouping/classification based on an attribute, here it is colour. Remember as the child sorts based on the colour name, child must be knowing the name. In case if the child is too young, then you can show blue colour card / paper and then ask the child to sort out the cars looking at the blue card/paper. Activities should match the developmental stage of the child.

Example for sorting.(1) sorting out blue cars



(the above group has 3 blue cars , 2 red cars and 1 yellow car)

Sorting is an important skill for preschool aged children to learn and practice. You may have noticed that preschoolers have a natural tendency to sort things. Children will begin to sort objects into groups. Sorting is a way to organize and make sense of their environment. Activities like these provide concrete objects for children to sort and organize. Colors, shapes and sizes are all great attributes to sort by but for older or more advanced learners try sorting by more than one or more subtle attributes. They may also sort things based on number, size, texture, and hardness. Eventually, they can sort based on other characteristics like mass, odour, sound, taste, and flexibility. By encouraging sorting, you are helping develop mathematical thinking and science reasoning skills

Let's examine the benefits of this activity.

- Fun, interactive and motivating!
- Great for fine motor development and hand/eye coordination.
- Self-correcting and problem solving.
- Language skills: naming the colour and pronunciation.
- Reinforce and extend learning about colours.

Example for sorting (2) Sorting Leaves and Seeds

Collect leaves, seed pods, and flowers in a large basket. Encourage children to sort out the collection. Talk about what children are doing. Encourage them to verbalize their ideas. Sorting can also lead to

graphing as well. Notice picture below? This picture graph shows what has been sorted and how many are there in each group.

Ideas for teachers for sorting activities.

What we use for sorting all depends upon the age and ability of the child, as well as their interests. We often don't need to buy anything fancy either! If we have "real" items, such as animal figurines or food, these should typically be used rather than magazine cut-outs or photographs. Sorting which allows the child to use their hands to handle real-life objects is much more beneficial for all-around learning.

- 1 sorting colour papers
- 2 sorting colour beads
- 3 sorting shapes
- 4 sorting food items which can be eaten raw
- 5 sorting leaves by shape
- 6 sorting buttons / balls by size
- 7 sorting objects by weight
- 8 sorting by taste etc.

But why do we need to develop sorting skills?

Sorting is the next step in the process which begins with matching. When a child puts two objects together according to a particular feature they are matching. Once they group a number of objects together they are sorting.

When children sort objects they are learning that some of those things are alike and some of those things are different. Once they move onto grouping according to a number of different attributes they learn that a single item can have many features by which it can be sorted.

This process of looking at an item, assessing its various characteristics and then organizing it with other items of the same characteristics is the first step in applying logical thinking to objects. This skill can then be applied to both mathematical concepts as well as to daily life.

why sorting is an important skill in math

Of the five primary strands of mathematical discipline, sorting falls under algebra which encompasses the understanding of patterns and relationships.

So, a child who has developed strong sorting skills will find it easier to:

- Make matches
- Identify sets

- Classify items by single attributes
- Classify items by multiple attributes
- Recognize and create patterns
- Understand patterns, relations, and functions
- Compare sets for differences and similarities
- Recognize relationships between sets
- Understand how rules apply to sets
- Meanwhile, a child who has not developed robust sorting skills may not only have difficulty later in math but also in:
 - Understanding how to connect new pieces of knowledge with what is already known
 - Making informed judgements
 - Making and enacting decisions
 - Coping with events that are out of routine
 - Dealing with the unexpected.

A child who has good sorting skills can:

- understand patterns, relations, and functions
- make matches
- identify sets
- sort and classify by various and multiple attributes
- compare sets for similarities and differences
- recognize and create patterns

3.6.2 Classification:

Next comes **classifying**. When learning to classify, children first learn how to classify by naming attributes that allow items to fit within a group, then move on to identifying attributes that exclude from a group.

Classifying on the other hand requires children to discover how a given set of objects might be grouped. The children are not told to put objects into groups based on a particular grouping or attribute as the case with simple sorting tasks. With classifying tasks children are provided a set of objects and then asked to group the objects based on their thinking as to how the objects in each group might be the same.

Classification or categorizing is integral to knowing and using number sense. Different ways to categorize and classify must be known to understand different ways a group or groups can be made

before understanding how to represent them numerically, as a quantity (cardinality) or other representation. Classification is seen in children's play as they sort, organize, group and regroup objects. Classify by one property (or attribute) using general properties and specific properties. Selecting objects using classification to duplicate a pattern. Get confused when objects have multiple properties with which they are classifying and will eventually learn to classify groups (sets) of objects by multiple attributes (properties) and represent them with Venn diagrams

The preschool programme provides various opportunities for children to develop the skill of classification / grouping ...

- Objects, organisms, events, and systems can be organized into groups with similar properties (classification).
- Classification is one way to organize objects, events, and ideas. (organization)
- Objects have properties.
- Objects are identified by their common properties.
- Objects can be grouped by external properties, color. (3.5 years)
- Objects are classified by their common properties.
- Objects have more than one property. (4.5 years)
- Objects in a group can share some characteristics while differing in others. (4.5 years)
- Objects, organisms, events, ideas, and systems can be organized into groups with similar properties.
- Objects are identified by names.
- Objects with similar properties are the same.
- Objects with different properties are different.
- Objects can have properties that are the same and different, but still be the same (triangles - same shape, different size, color)
- Objects can be grouped (classified) into sets/ groups
- Sets can have cardinality. Cardinality of sets can be the same or different.
- Objects with similar properties that change sequentially can be ordered by that property.

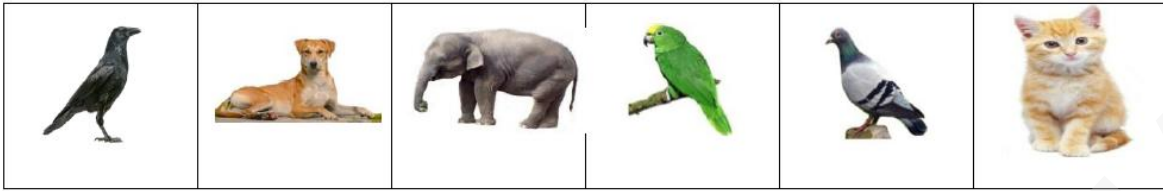
Example for Classification

3.6.3 Classification (Birds and Animals)

Material Required: Six picture cards

Instructions:

1. Place all the picture cards of birds and animal in front of the child in a random order. Now, ask the child, 'Pick out the picture cards of all the birds from the pile'. Encourage children to explain the characteristics on which they have done the classification.



3.6.4 Identification and Classification (Colour)

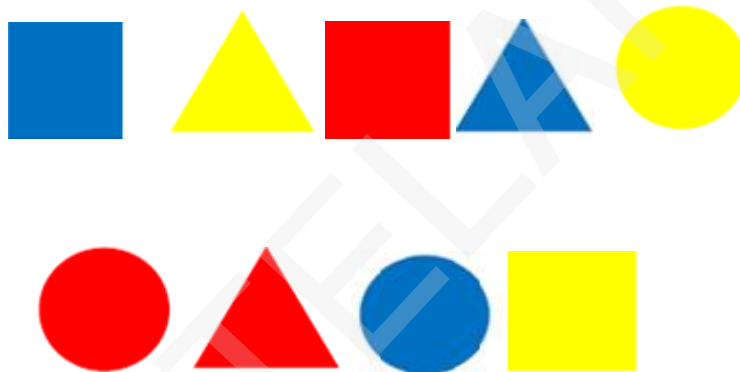
Stage 1

Classification based on one attribute (colour)

Materials Required: Nine big cut outs of different colour and shape cutouts.

Instructions

- Spread nine big cut outs of different colour and shape in front of the child in random order in a tray and ask the child, **‘Take out all blue colour cut outs from the tray.’**



Here the child is asked to classify the shapes based on colour (one attribute) .The prerequisite for this activity is that child should have acquired the skill of matching based on colour and also identify the names primary colours.

3.6.5 Identification and Classification (Shape)

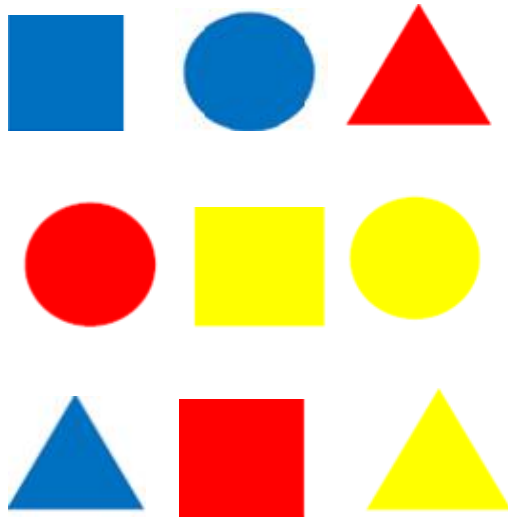
Stage 1

Classification based on one attribute(shape)

Materials Required: Nine big cut outs of different colour and shape cutouts.

Instructions

- Spread nine big cut outs of different colour and shape in front of the child in a random order in a tray and ask the child, **‘Take out all square shape cut outs from the tray.’**



Here the child is asked to classify the cutouts based on shape (one attribute) .The prerequisite for this activity is that child should have acquired the skill of matching based on shape and also identify different basic shapes.

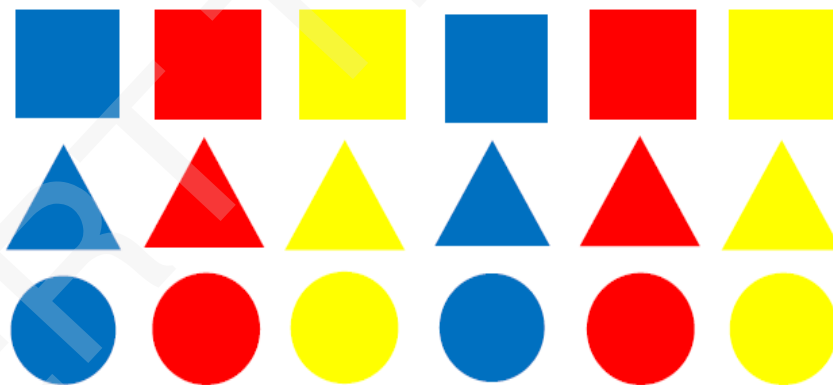
Stage 2

3.6.4 Classification based on two attributes (shape and colour)

Materials Required: Eighteen big cut outs of different colour and shape cutouts.

Instructions

- Spread eighteen big cut outs of different colour and shape in a random order in a tray and ask the child, ‘Take out all blue square cut outs from the tray.’



Here the child is asked to classify the cutouts based on shape and colour (two attributes) .The prerequisite for this activity is that child should have acquired the skill of matching based on colour and shape and also identify different basic shapes and colours.

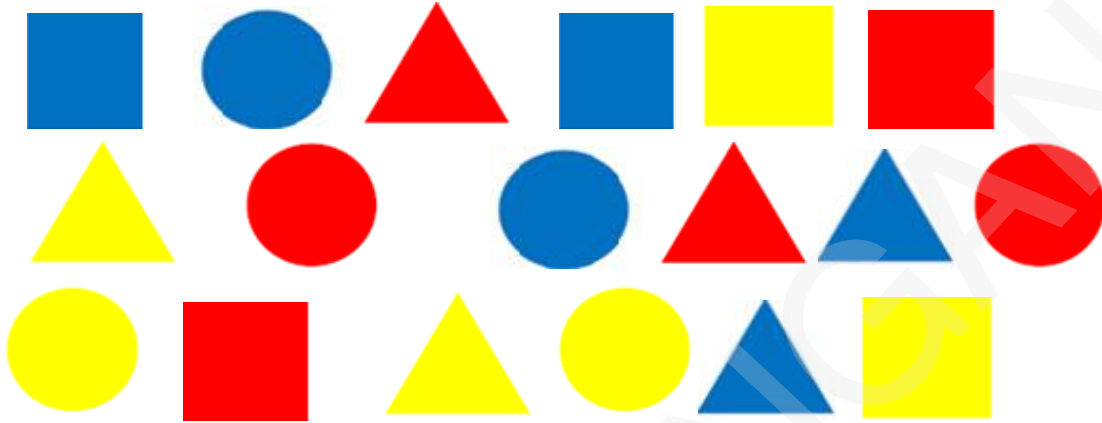
Stage 2

3.6.6 Classification based on two attributes (shape and colour)

Materials Required: Eighteen big cut outs of different colour and shape cutouts.

Instructions

- Spread eighteen big cut outs of different colour and shape in a random order in a tray and ask the child, **'Take out all yellow triangle cut outs from the tray.'**



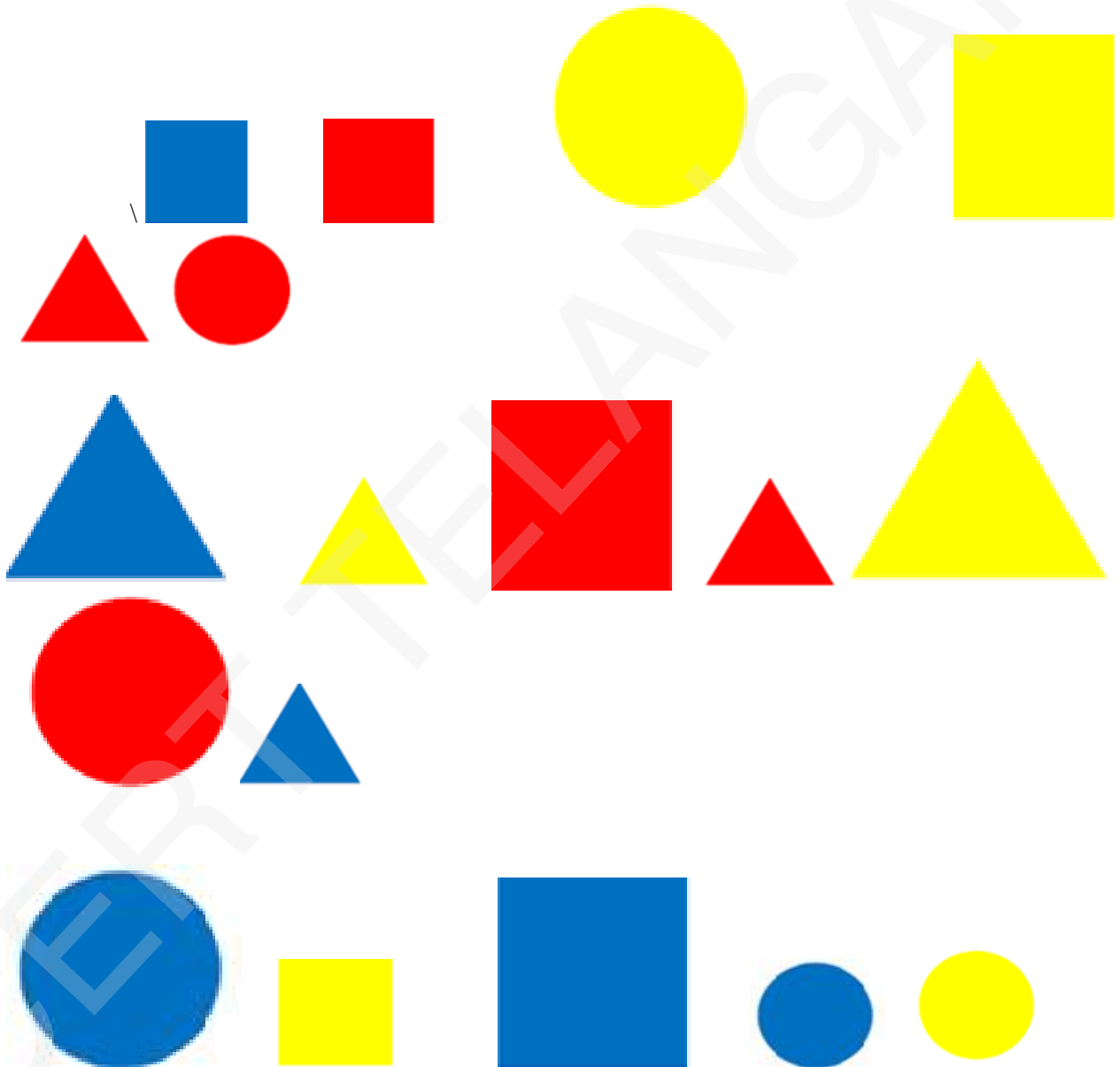
Here the child is asked to classify the cutouts based on shape and colour (two attributes) .The prerequisite for this activity is that child should have acquired the skill of matching based on colour and shape and also identify different basic shapes and colours.

3.6.7 Identification and Classification of Colour, Shape & Size

Materials Required: Nine big cards of different colour and shape, nine small cards of different colour and shape

Instructions:

- Spread nine big and nine small cut outs of different colour and shape in a random order in front of the child and ask the child, **‘Take out the big red square cut out and give it to me’**



Here the child is asked to classify the cutouts based on size ,shape and colour (three attributes) ie BIG , RED and SQUARE.The prerequisite for this activity is that child should have acquired the skill of matching based on size , colour and shape and also identify sizes , different basic shapes and colours.

Plan classification activities ..

- Bring samples of food articles distinct taste for the children, ex. Lime, imli, sugar, salt, neem etc. let each child try and identify the different taste and associate them with the food articles.
- Make some picture cards of food items with distinct taste. Let the children classify these as salty, soar, sweet and bitter.
- Make simple classification cards. These could have any one shape in different colours. Let the children put them together cards of the similar.
- Make classification cards involving to attributes, for eg. The cards may include three shapes and three colours. Let the children categories them in any way they want. For the older age group more than two attributes can we involved. For eg. Size, shape and colour. Children can classify the cards now in three ways.

Summing Up

Remember, which ever material you are using should be appropriate for the concept /age /culture. Always first teacher should demonstrate the activity involving all children then teacher should provide opportunities for all children to do on their own and while doing the activity teacher should be there and ask questions which makes children to think and answer logically. This leads to the mathematical thinking in young children.

Classifying and sorting activities help children to develop a range of thinking skills and build the foundations for later problem solving. The visual memory and discernment involved, and the ability to identify patterns, relationships, similarities and differences, assists children in learning about early number representation and problem solving.

Classifying and sorting activities can also be good for developing fine motor skills. Fine motor skills help children engage with the smaller more precise movements they will need as they continue to grow and develop. Many classification and sorting activities lend themselves to being great fine motor activities as well.

Unit end questions:

- 1) Write the importance for developing classification skill in preschool stage.
- 2) Plan and prepare worksheets for preschool ,children in then following ..
Identification of similarities and differences in the given pictures.

UNIT 4 - COMPARE, ORDER / SERIATE & MEASURE

4.0 OBJECTIVES

After completing this unit as a preschool teacher trainee, you will be able:

- To differentiate between perceptual knowledge and conceptual understanding among children
- To explain subitising in math context
- To develop measuring abilities among children through activities
- To compare and contrast between different types of measurement
- To differentiate between non-standard and standard tools for measurement
- To state the history of metric system that evolved from French revolution
- To use both non-standard tools and standard tools in your classroom activities

4.1 INTRODUCTION

Over the years of research we could envisage that children with well-developed numerical abilities during their early years are more likely to have mathematical success over the long term.

In this chapter we will understand how we can develop sensitivity among children and move them from perceptual knowledge to conceptual understanding and subitising.

4.2 DEVELOPING SENSITIVITY AMONG CHILDREN & MOVING FROM PERCEPTUAL KNOWLEDGE TO CONCEPTUAL UNDERSTANDING

When we glance outside we find several observable facts in the world at large. Most of the time consciously or unconsciously we start to compare, we try to put in sequence, we classify and sometimes we measure. This is because we know subitising and this is a big skill for children. In a simple way we say that subitising is an ability to quickly look at few items in a box and able to identify the number of items without counting them in a small set. An example of subitising is to look at the picture below and say 5 sweets without counting.



Fig.1

Subitising to say it has 5 sweets

Subitising is of two types namely perceptual and conceptual. Perceptual come from our perceptions which means our ability to be aware of our surrounding through the senses. Conceptual comes from concepts or abstract ideas. Let us learn this with an example.

Perceptual knowledge	Conceptual Understanding
<ul style="list-style-type: none"> • Earth is flat • Earth does not move • Sun moves around the earth 	<ul style="list-style-type: none"> • Earth is not flat but is round • Earth moves • Earth also moves around the sun

Perceptual knowledge is straightforward way of our perception whereas conceptual involves abstracts ideas.

To teach kids subitising we may use dot cards to match the numerals. Fig.2 to be matched with Fig.3

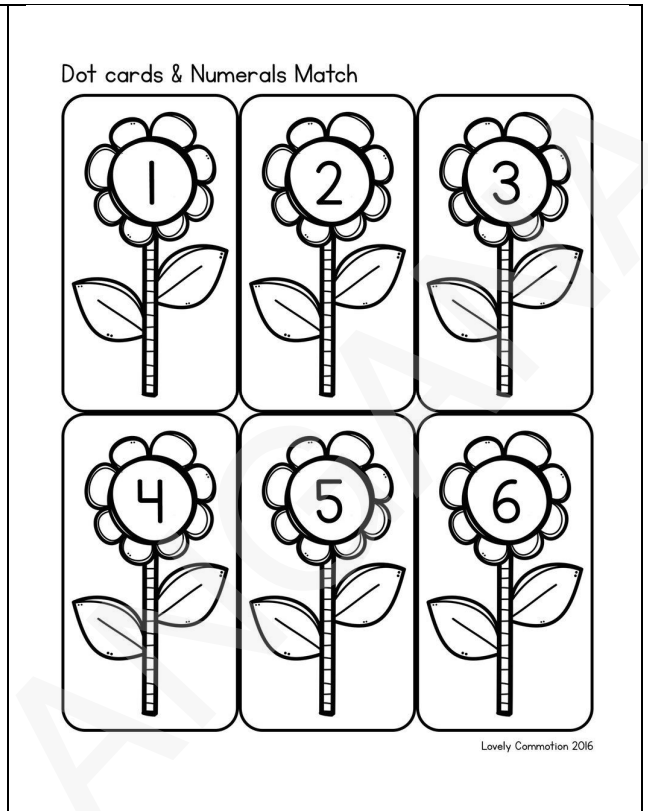
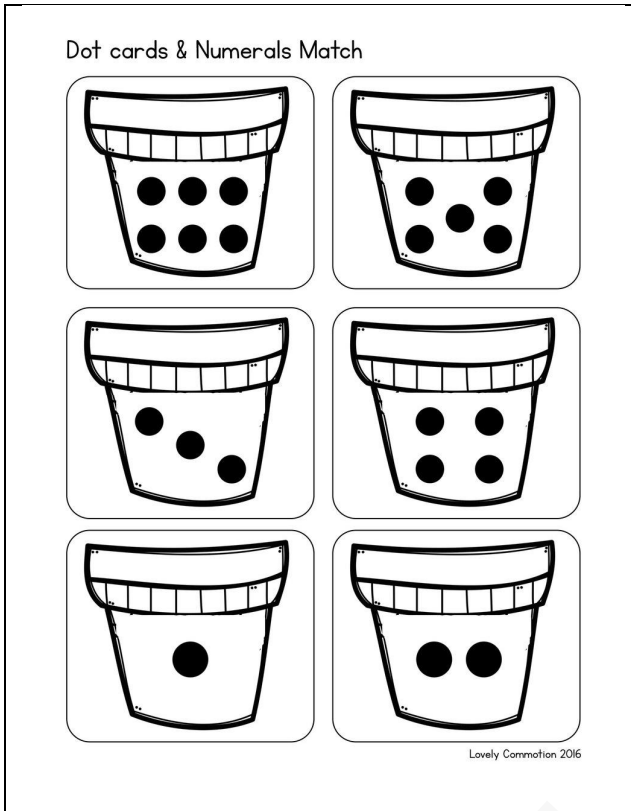


Fig.2
Teaching subitising with dot cards to match numerals in Fig.3

Fig.3
Match with Fig.2

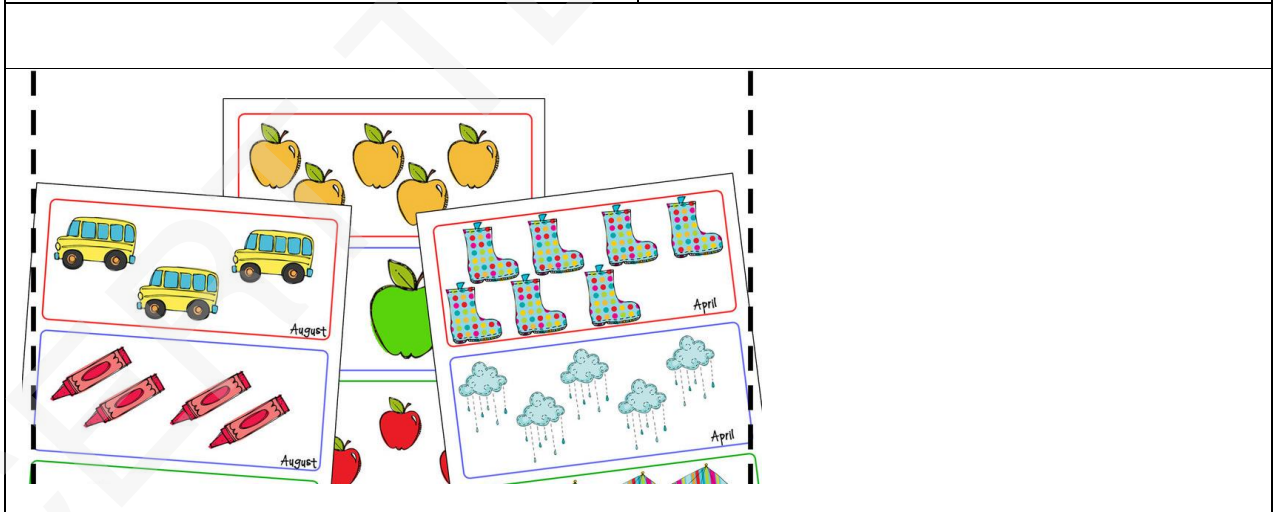


Fig.4 Other cards that can be prepared to teach subitising

Let us understand the difference between perceptual and conceptual understanding:

- ❖ Both perceptual and conceptual subitising are cognitive processes
- ❖ Perceptual comes on the basis of perception or sensation made by us
- ❖ Conceptual understanding is a quality that only humans possess
- ❖ Conceptual and perceptual processes go on into our brain at the same time though by different parts of the body

To develop conceptual understanding among children we need to teach them the concept of measurement.

4.3 DEVELOPMENT OF MEASURING ABILITIES THROUGH ACTIVITIES

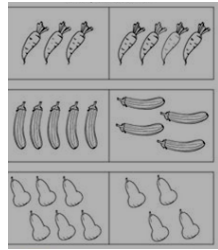
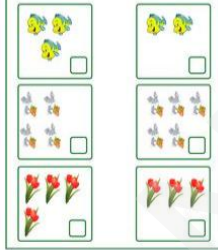
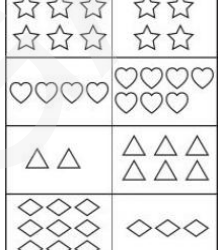



The action of measuring length, speed, temperature, weight, time etc. is called measurement. Measurement is a part of our daily routine. We measure size of our footwear, height and weight of a child, the food that is approximately given in the tiffin box, time taken to finish a task, counting to measure number of pencils etc.



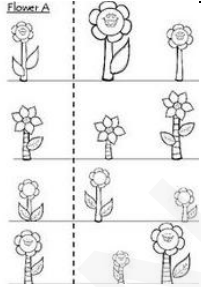
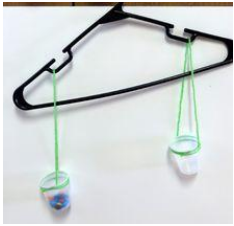


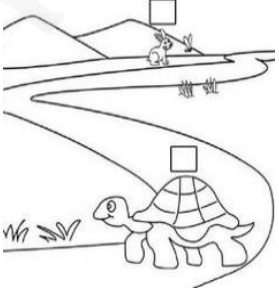


Measurement skills are learnt through experiences and opportunities that children learn in their daily life. This can be done by:

- Comparing
- Ordering
- Measuring

4.3.1 COMPARE:

The literal meaning of comparing is to find out the similarities and differences between two or more objects/ items. Children need to understand the following related to comparison:

Comparing concept	Activity 1	Activity 2	Activity 3
More – Less			
	<p>Fig.5</p> <p>Understanding more or less in vegetables</p>	<p>Fig.6</p> <p>Tick the box that has more objects</p>	<p>Fig.7</p> <p>Circle the group that has less</p>
Big - Small			
	<p>Fig.8</p> <p>Circle the BIG picture</p>	<p>Fig.9</p> <p>Circle the SMALL picture</p>	<p>Fig.10</p> <p>Flash cards to teach big-small</p>


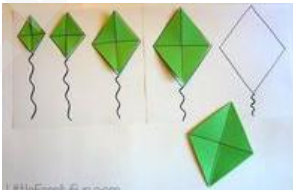

Tall-short			
	<p>Fig.11</p> <p>Circle the shorter image</p>	<p>Fig.12</p> <p>Number the tallest as 1, second tallest as 2 and shortest as 3</p>	<p>Fig.13</p> <p>Circle the tall flower</p>
Heavy-Light			
	<p>Fig.14</p> <p>Demonstrating heavy and light objects in the class by using stone and flower</p>	<p>Fig.15</p> <p>Circle the heavy object</p>	<p>Fig.16</p> <p>Circle lighter and heavier objects</p>
Far-Near			
	<p>Fig.17</p> <p>Finding far and near animals</p>	<p>Fig.18</p> <p>Finding the animal that is closer to the tree</p>	<p>Fig.19</p> <p>Demonstrating Near and Far in the garden</p>




Children should be taught vocabulary related to size, distance, weight through comparisons.

Similar to the above, plan activities on the concepts related to smaller-larger, more than-equal to-less than etc.


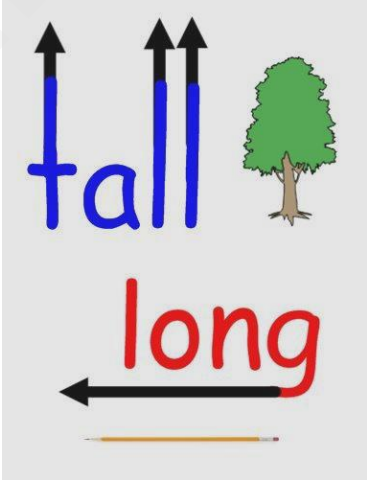
4.3.2 ORDER (SERIATION):

Ordering means arrangement based on a method / strategy / particular sequence / pattern. Children need to understand the following related to ordering:

Comparing concept	Activity 1	Activity 2	Activity 3
Smallest Biggest			<p>*</p> 
	<p>Fig.20 Identifying the smallest</p>	<p>Fig.21 Identifying the biggest</p>	<p>Fig.22 Cards to match biggest and smallest</p>

Heaviest -Lightest			
	Fig.23 Weighing the stones to find heaviest and lightest	Fig.24 Taking items from classroom to see the heaviest and lightest	Fig.25 Comparing heaviest and lightest animals

It is a responsibility as a teacher to teach children the differences between long / short and tall short. Let us see this with some examples:









	
Fig.26 & 27 Height wise measurement is tall and width wise measurement is long used in different contexts	

4.4 MEASUREMENT:

Children at this age may not be developmentally ready to learn about formal measurement. Measurement can be taught using *non-standard tools and standard tools*.

4.4.1 NON-STANDARD TOOLS FOR MEASUREMENT:

Let us begin our discussion by using non-standard tools like wooden strip, Lego blocks, hand span, thumb width, cubes, pencil, eraser, paper clips to measure things. These are called non-standard as they are not uniform and differ from person to person or object to object. Example: A person's hand span may differ based on the size of his/her hand.

Objects to Measure	 Cube Prediction	 Cube Measurement	 Paper Clip Prediction	 Paper Clip Measurement
 Pencil		My pencil is _____ cubes long.		My pencil is _____ paper clips long.
 Crayon		My crayon is _____ cubes long.		My crayon is _____ paper clips long.
 Book		My book is _____ cubes long.		My book is _____ paper clips long.
 Hand		My hand is _____ cubes long.		My hand is _____ paper clips long.






Fig.26 Measuring using non-standard units	Fig. 27 Measuring using Lego Blocks
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<p>Fig.28</p> <p>Measuring using Hand Length</p>	<p>Fig.29</p> <p>Measuring pencil using paper clips</p>

These help the little ones to grasp the concept of measurement using non-standard tools and how to use them correctly. They may not learn these in a day or a first attempt. The teacher needs to give them lots of practice and repeat the activities so that they learn how to measure correctly. Once they master this process, we need to introduce them to standard units of measurement.

4.4.2 STANDARD TOOLS FOR MEASUREMENT:

When we say we are measuring through standard units, we mean we are using standard tools like ruler, tapes and other measuring devices that give us exact results irrespective of the person who is measuring. The results for length are usually given in inches, feet, yards, centimeters, millimeters etc.

The history of Standard units of measurement dates back to **French Revolution**. We will learn about metric system and how it moved to SI units.

Gavin Kirby in 2014 in his article “**The History of the Metric System: from the French Revolution to the SI**” stated the following:

“The story of the metric system starts, somewhat unexpectedly, in 17th Century England, with the proposal by the natural philosopher John Wilkins for a consistently decimal system of units. The Europe

of his time was divided by the use of innumerable different systems of measure, with each country using its own traditional units. To remedy this, Wilkins conceived of a single, rational standard, based on the best science of his time, which all nations could equally share. His idea for a “universal Measure” was not adopted in his era. His proposed standard unit of length was defined by the length of a pendulum with half-period equal to one second; this length is extremely close to the modern definition of the metre.

During the Age of Enlightenment, with the growing momentum of scientific progress and the ever-greater necessity of international collaboration, the lack of a true standard of measurement became an increasing problem.

However, it was only in France that the idea of the metre first encountered fertile cultural soil in which it could take root. The French scientists of the late 18th Century created the first practically-implemented version of the metric system. In the words of the philosopher and mathematician Condorcet, it was intended to be “for all people, for all time: a system that was defined in a logical and abstract mathematical way, not with respect to authority, tradition, local custom or human anatomy.

By the end of that century, the utility of the metric system had been widely recognised, and it had become the official measurement system of virtually all of continental Europe.

In the English-speaking world, by contrast, the new measures were much slower to take hold, initially because of the apathy of the British Empire and its hostility to the French Revolutionary and Napoleonic regimes. However, by the late 19th Century the great utility of the metric system in science and international commerce had become impossible to ignore, and in 1875 the major world powers, including the United States, signed the Treaty of the Metre, which gave the metric system its first official recognition as the international standard of measurement.

Despite this, the British Empire still refused to adopt the new measures as its primary system, and thus a large part of the potential benefit of these newly-created standards was lost.

During the 20th Century, the metric units continued to become more widely adopted, most notably in science, a trend present even in the English-speaking world.

The most important step in the development of the metric system in the 20th Century was the creation of the International System of Units (i.e. the SI) in 1960: this was a rationalised version of the metre-

kilogram-second (MKS) system, and it rendered obsolete the older CGS (centimetre-gram-second) metric systems. This took place in the age of reconstruction after World War II, a time in which international organisations such as the United Nations were being established, and the importance of international cooperation was becoming more widely appreciated. The SI was intended to become the sole system of measurement on Earth, a logical mathematical language that all nations could share alike, replacing the various systems of customary units that were still in use throughout the world, most notably in the English-speaking world.”

India too had traditional methods to measure lengths during olden times. They were using anguli (literally finger), haath (elbow to end of middle finger) and gaz but the problem was these measures were not properly standardised and differed from region to region. Later India has officially adopted to the metric system of measurement (in December 1956 with the *Standards of Weights and Measures Act*, which took effect beginning 1 October 1958) also known as International Systems of Units (SI) that was originated in France. Most countries across the world follow the SI system except 3 countries namely United States, Liberia and Myanmar. This is the system used for all purposes of global trade.



Fig.30

Standard tools for measurement


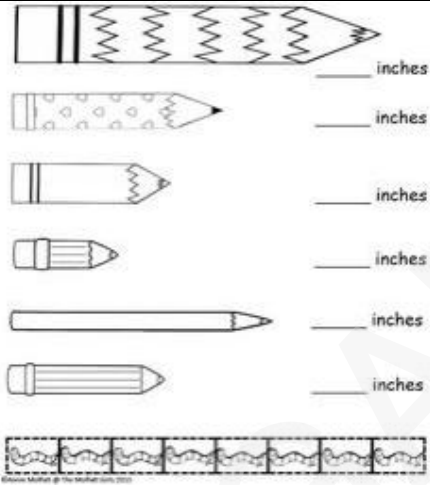

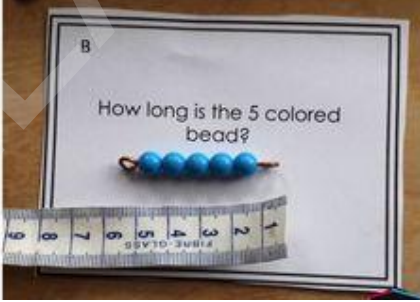
Now let us come back to standard measuring tools. As a teacher we should start introducing to standard tools when the child understands hand spans and other non-standard measurements. This is generally after the age of 4 years. Start introducing smaller measurements of length on a ruler. Make them aware that irrespective of the person who measures it, the result remains the same.

Children may not be right in the first go. For example when a child is measuring using a ruler, s/he may not place the initial point at zero or may not hold the ruler straight. The child finds these aspects a bit difficult. The teacher needs to encourage the child to learn how to measure correctly giving iterations of the same concept using different material.

You may teach children the following before you start with activities

Unit	Tool	Illustrations that are approximate to the unit
Millimeter	Scale / ruler	<ul style="list-style-type: none"> • Thickness of a staple pin (1mm) • Sharpened pencil tip (1mm)
Centimeter	Ruler / Measuring tape	<ul style="list-style-type: none"> • Length of a staple pin (1cm / 10mm) • Width of a pumpkin seed (1cm) • Width of a small peanut (1cm) • A regular peanut is 2cm, Walnut 4cms, Lemon 5cms
Inch	Ruler / Measuring tape	<ul style="list-style-type: none"> • Size of a Pepsi/ Fanta bottle cap (1 Inch / 2.54cms)
Feet	Measuring tape	<ul style="list-style-type: none"> • Length of a hammer (1 foot / 12 inches) • Sofa is 6 feet
Yard	Yard stick	<ul style="list-style-type: none"> • Length of a door (1 yard / 3 feet / 36 inches)

The activities that you can plan to teach standard measurements:

	
<p>Fig.31</p> <p>Using measuring tape to measure the blocks</p>	<p>Fig.32</p> <p>Using inches ruler to measure the pencil</p>
	
<p>Fig.33</p> <p>Using measuring tape to measure the height of a boy</p>	<p>Fig.34</p> <p>Measuring using tape</p>

Plan activities to teach how to measure other objects in the class in standard units.

Similarly mass i.e. weights can be taught (in milligrams, grams and kilograms) and liquids can be taught using measuring cups (in milliliters, litres and kilolitres) with the steady mental development of the child over the years.

4.5 SUM UP:

Learning to measure is a fun for children. With day to day activities and experimentation it may be observed that children learn to understand the concepts of measurement. In this unit, we have seen how to develop sensitivity among children and move them from perceptual knowledge to conceptual understanding using subitising. We have also learnt about developing measuring abilities among kids through classroom activities that include comparing, ordering and measuring. Measurement can be initially introduced using non-standard tools and then moved to standard tools. The teacher is also made familiar with history of metric system to SI units (International standard units of measurement) taking into account the French revolution.

With this knowledge as a teacher we may start to work on the children so that they begin to explore and compare between different items, learn how to put them in a sequential order and discover how to count and measure objects so that they become better problem solvers.

4.6 QUESTIONS:

1. How to move children from perceptual knowledge to conceptual knowledge? Explain with examples.
2. What are the tools for measurement? Give illustrations and plan activities for the same.
3. How is the history of metric system from French revolution useful for a early childhood teacher?
4. What are nonstandard tools for measurement? Why should they be introduced to the children?

UNIT 5 - DEVELOPING NUMBER SENSE UP TO 10

5.0 OBJECTIVES

After completing this unit as a preschool teacher trainee, you will be able:

- To understand the importance of number sense in broad terms
- To describe the need for developing number sense
- To discover the need for foundational number sense
- To apply different principles to develop number sense in your classroom
- To design activities for building number sense in children
- To create worksheets to develop number sense

5.1 INTRODUCTION

Today's children are bonded to technology and tech gadgets. A more than one year old child understands the usage of mobile and its various applications. When you ask a child to give back the mobile or a gadget while they are using, they simply reply '2 minutes'. They are trying to imitate elders, though they do not understand what exactly is '2 minutes' but they know that once it's been told, the elder will definitely wait and they could finish watching/playing with the gadget. This is just an instance; likewise children use numbers in many ways in their daily routine well before they join school.

The first step in the direction of getting kids to build sense of numbers is to see numbers as a sense-making tool. Begin to use math in life, such as counting and giving them biscuits or toffees or any other item. Understanding on how math is used in daily life through precise examples helps kids know why the numbers are so important. The real-world applications make them feel more excited to learn about math.

5.2 NUMBER SENSE IN BROAD TERMS

Number sense may be stated as follows:

- associating numerical symbols with quantity
- understanding the values of numbers in relation with other numbers
- use of numbers in a flexible way while using various fundamental mathematical operations
- development of various strategies to count, measure and estimate

5.3 WHY NUMBER SENSE NEEDS TO BE DEVELOPED?

A child having well developed number sense in his/her early childhood years is a good performer in the later years of life. Over the years learners comprehend that there is a flexible approach to working with numbers, start to work on numbers in their brains and learn to manipulate with numbers to improve on the mental math abilities.

5.4 FOUNDATION FOR NUMBER SENSE

Mostly we see that foundation for number sense begins at home. When a mom asks a child to pick up 2 biscuits from a container, the child picks up only 2 biscuits. The child understands the language that is spoken at home. The child is in a position to attach the meaning and begins to understand different numbers with values. It may be noticed that by the age of 3 most children will be able to clearly distinguish a box containing more chocolates and another box containing less chocolates. This foundational number sense plays a very important role for development of mathematical skills over the years.

According to **Judy Sayers, Paul Andrews, and Lisa Björklund Boistrup** of Stockholm University during 2016 in their study “The Role of Conceptual Subitising in the Development of Foundational Number Sense” found eight components for foundational number sense and they summarized them as follows:

1. Number recognition

Number recognition entails being able to recognise number symbols and know associated vocabulary and meaning. It entails being able to identify a number symbol from a collection of number symbols and name a number when shown that symbol.

2. Systematic counting

Systematic counting includes ordinality and cardinality. Children should be able to count to twenty and back or count upwards and backwards from any starting point, knowing that each number occupies a fixed position in the sequence of all numbers.

3. Awareness of the relationship between number and quantity

Children who are aware of the relationship between number and quantity understand the one-to-one correspondence between a number's name and the quantity it represents - that the last number in a count represents the total number of objects.

4. Quantity discrimination

Quantity discrimination includes an awareness of magnitude and of comparisons between different magnitudes. Children can deploy language like 'bigger than' or smaller than'. Children will have moved beyond counting as a mechanical routine.

5. An understanding of different representations of number

Includes an understanding that numbers can be represented differently. Such representations can include a number line; different partitions of a number; the use of fingers and various manipulatives.

6. Estimation

Children should be able to estimate, whether it be the size of a set or an object. Estimation involves moving between representations of number, for example, placing a number on an empty number line.

7. Simple arithmetic competence

Children should be able to perform simple arithmetical operations or the transformation of small sets through addition and subtraction.

8. Awareness of number patterns

Includes awareness of number patterns and, in particular, being able to identify a missing number.

Based on their systematic analysis of the literature they identified eight distinct but not unrelated characteristics of foundational number sense.

How will you apply these principles in your classroom?
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5.5 HOW TO DEVELOP NUMBER SENSE?

In order to develop number sense we need to build in children the concrete level of mathematical understanding which is the most basic level of understanding. This is important as well as very crucial level to widen math concepts and also conceptual understanding. The child is able to learn when s/he is provided with plenty of opportunities to manipulate concrete-objects to solve problems. Here are some of the ways in which number sense can be developed in children.

Begin introducing numbers through Rhymes

<p>Rhyme 1</p> <p>One, two, three, four; Mary's at the cottage door, Five, six, seven, eight; Eating cherries off a plate.</p>	<p>Rhyme 2</p> <p>One, two Buckle my shoe, Three, four Open on the door, Five, six Pick up sticks, Seven, eight Lay them straight, Nine, ten A good fat hen</p>
<p>Rhyme 3</p> <p>One, two, three, four, five, Once I caught a fish alive, six, seven, eight, nine, ten, Then I let it go again.</p> <p>Why did you let it go? Because it bit my finger so. Which finger did it bite? This little finger on the right.</p>	<p>Rhyme 4</p> <p>Ten little fingers, ten little toes, Two little ears and one little nose Two little eyes that shine so bright And one little mouth to kiss mother goodnight.</p>

Number words in correspondence with items

Encourage the child to count slowly and steadily each object *only one at a time* so that they learn to develop one-to-one correspondence with the numbers. Bring in the thought that each item in the given set of items must have a unique count word.

Begin with numbers up to 5. Show them some 4 books or toys. Assign a number to each of them in a group of 4 books or toys. In the same way cheer them up for every correct instance.

Slowly increase the number up to 10.

Train on 'keeping track' approach

Gather children and begin to count loudly. When children are observing, make sure you physically move the items as you count loudly and ask them to repeat.

Converse on how the last number mentioned stand for number of objects in the group altogether

Give confidence to the children to count and ask them to stop counting when the items in the set are exhausted. The last number counted gives them the total number of items in the set known as cardinality.

Pick few students from the class and ask them to stand in a line. Pick a child to start counting those standing students. Once the child finishes counting the last student, say loudly "6 students in all". Ask the children in the class to repeat and understand that, number of students who are picked up by the teacher to stand are 6 in number.

Provide chance for children to produce sets of a given size

Give a target number for the children and ask them bring in the target number of objects. For example ask a child to get 6 pencils from his friends in the class. If the child gets the target number of pencils then applaud him. If he does not, then remind him of the target number given and ask him to recount and replay till he gets correctly.

Teacher telling a story and child doing a picture art

A teacher should describe a story involving numbers. Let us say:

On a Sunday evening Rahul went to a garden near his house. He saw sun setting in orange and yellow colour. The children listen to this and draw 'one' sun on their paper. Rahul saw 2 black cats running around and playing. Then children draw 'two' cats. There were three big green trees. Now children draw 'three' trees and this goes on....

The children listen to the numbers and draw the same number of objects and colour them.

Choose a Number for a day

Along with children choose any number from 1 to 9 and start involving the number all through the day. Say you have chosen number '7'.

- You may ask children to circle 7 in the calendar
- Children may be asked to check number 7 in their books
- You can relate facts associated with number 7 – there are 7 days in a week, a rainbow has seven colours, 7 wonders of the world etc.
- You can ask them to cut 7 pieces of paper
- You may ask them to count and place 7 objects in a box

This enables them to comprehend the number and its value.

Understanding the pattern of dots in a dice

Start playing games like 'Snake and ladder' or 'Ludo' with dice so that child looks at the pattern of the dots, their arrangement and is able to get a grasp of different numbers. The child may use the given cards or paper cut outs with number on it, the child may be asked to place bindis stickers according to the number on the cards etc. Similar activities may be planned to associate number with dots and vice-versa.

Play on 'estimation'

Show very few objects (say 3 or 5 or 6) to the kids for 5 to 10 seconds and then cover them up with a veil (cloth). Now, ask the children to think and state the number of hidden objects. This helps children to estimate and it also challenges their mental thought process.

Games

Playing games is fun and children love to play games. So bring in the concepts of counting through games.

Based on the research of **Gwen Dewar** on **Preschool number activities: How do you introduce numbers to preschoolers?** The following principles involving games encourage kids to think about several key concepts, including:

- The one-to-one principle of numerosity (two sets are equal if and only if their items can be placed in perfect, one-to-one correspondence)
- The principle of increasing magnitudes (the later number words refer to greater numerosities)
- The one-to-one principle of counting (each item is to be counted is counted once and only once)
- The stable order principle (number words must be recited in the same order)
- The cardinal principle (the last word counted represents the numerosity of the set)

Illustrations:

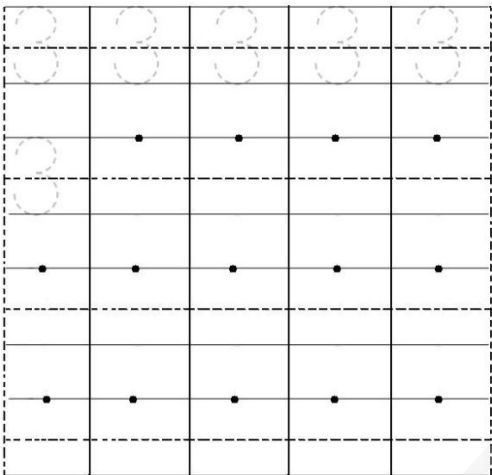
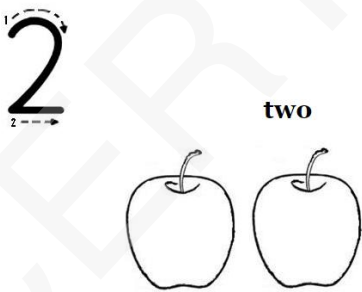
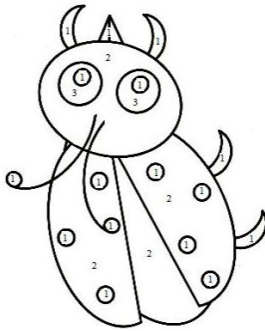
You may begin conversing in classroom and work on the following in a play way method:


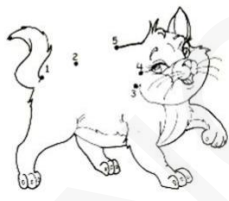
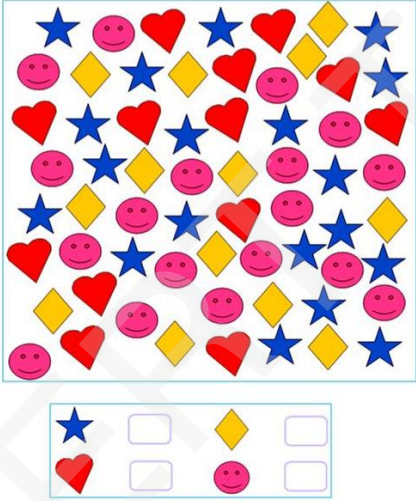

- ✓ Children may be given five green paper glasses and some yellow paper glasses. Ask children to place five green glasses on the table. Corresponding to each green glass a child is supposed to pull out a yellow glass and place next to the green glass. This way they learn the one-to-one principle of numerosity.
- ✓ You may ask a child to get up and begin counting all the children wearing 'Red' coloured dress. Also, other colours may be given for other children. This way they learn the principle of counting.
- ✓ Children may be given some dot cards as train tickets. They should count the number of dots on their given cards. The number should be recited in the same order. If the child follows the same order and counts correctly the number of dots, the child's train ticket becomes valid to board a toy train. This helps them to learn the stable order principle.

Plan some more games based on the above principles.

5.6 WORKSHEETS TO BUILD NUMBER SENSE

Here are some of the worksheets that you can plan as activities to work in your class. You can create some more based on these thoughts shared.

	<p>Circle all the number 2's:</p> <table data-bbox="922 611 1242 1052"> <tbody> <tr> <td>7</td> <td>2</td> <td>2</td> <td>5</td> </tr> <tr> <td>2</td> <td>10</td> <td>2</td> <td>9</td> </tr> <tr> <td>6</td> <td>1</td> <td>9</td> <td>2</td> </tr> <tr> <td>4</td> <td>2</td> <td>2</td> <td>8</td> </tr> <tr> <td>10</td> <td>7</td> <td>3</td> <td>2</td> </tr> <tr> <td>1</td> <td>5</td> <td>2</td> <td>9</td> </tr> <tr> <td>6</td> <td>2</td> <td>8</td> <td>2</td> </tr> </tbody> </table>	7	2	2	5	2	10	2	9	6	1	9	2	4	2	2	8	10	7	3	2	1	5	2	9	6	2	8	2
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4	2	2	8																										
10	7	3	2																										
1	5	2	9																										
6	2	8	2																										
<p>Fig.1 Number tracing of number '3'. Plan for other numbers</p>	<p>Fig.2 Circle the number '2'. This gives them number recognition. Similarly plan for other numbers</p>																												
																													
<p>Fig.3 Number tracing of number '2' and colouring the objects</p>	<p>Fig.4 Colour identifying the number.</p>																												

<p>Plan for other numbers</p>	<p>Colour pink for 2</p> <p>Colour black for 1</p> <p>Colour 3 with yellow</p>
<p>Match the object with the correct numbers:</p> 	<p>Coloring Is Fun</p> 
<p>Fig.5</p> <p>Match the object with the correct numbers</p>	<p>Fig.6</p> <p>Connect the numbers and colour the picture</p>
	
<p>Fig.7 Count the number of objects and write the answers in the boxes</p>	<p>Fig.8</p> <p>Match objects to learn one-to-one correspondence</p>

	<table border="0"> <tr> <td>7</td> <td>Eight</td> </tr> <tr> <td>3</td> <td>One</td> </tr> <tr> <td>10</td> <td>Five</td> </tr> <tr> <td>5</td> <td>Seven</td> </tr> <tr> <td>9</td> <td>Four</td> </tr> <tr> <td>2</td> <td>Three</td> </tr> <tr> <td>8</td> <td>Six</td> </tr> <tr> <td>4</td> <td>Ten</td> </tr> <tr> <td>1</td> <td>Two</td> </tr> <tr> <td>6</td> <td>Nine</td> </tr> </table>	7	Eight	3	One	10	Five	5	Seven	9	Four	2	Three	8	Six	4	Ten	1	Two	6	Nine
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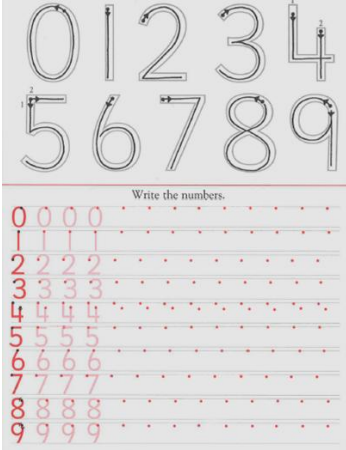
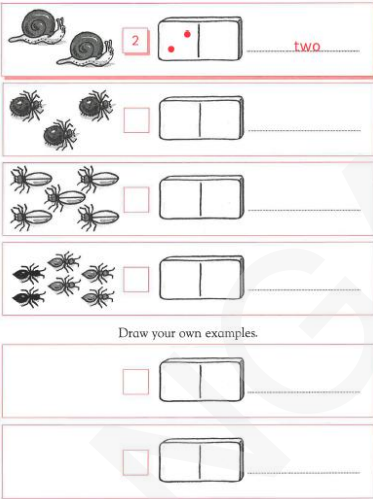
Fig.9
Number the picture as per the order

Fig.10
Match number with number name

	<p>One little two little three little Indians Four little five little six little Indians, Seven little eight little nine little Indians Ten little Indian boys.</p> <p>Ten little, nine little, eight little Indians Seven little six little five little Indians Four little three little two little Indians One little Indian boy.</p>
--	---

Fig.11

Fig.12

Fill the missing numbers	Rhyme for reciting numbers - forward and backward
	
<p>Fig.13</p> <p>Trace the numbers upto 9</p>	<p>Fig.14</p> <p>Count the insects, draw the dots and write the numbers</p>

Design few worksheets / activities to teach cardinality of numbers.

5.7 SUM UP:

The level of number sense has a big role in designing an educational setting for a child. The child's needs should be taken into account before orchestrating our teaching strategies and planning our learning environment. It is better for a teacher to firstly interact with the children, put them into groups before planning and implementing strategies and activities. There are several ways to teach number sense. We have seen some of them in this unit, we will see some more in the coming units.

In this unit we have learnt about:

- Number sense in broad terms
- Need for developing number sense
- Foundation required to build number sense

- Different ways to develop number sense
- Usage and creation of worksheets to build number sense

5.8 QUESTIONS:

1. What is number sense? How it can be developed?
2. Explain about foundational number sense?
3. State the cardinal number principles with examples.
4. Develop an activity to teach stable order principle in your classroom.

UNIT 6 - DEVELOPING NUMBER SENSE

PART A

6.0 OBJECTIVES

After completing this unit as a preschool teacher trainee, you will be able:

- To understand the importance of number sense and strategies used for developing number sense
- To apply strategies for developing number sense in your classroom
- To differentiate between counting, counting all, counting on, counting up
- To demonstrate the counting techniques in your classroom activities
- To develop among children visualization to make them solve word problems
- To utilize different ways of making children to work on addition and subtraction problems

6.1 INTRODUCTION

Children show keen interest and love for numbers and the basics start with reciting rhymes related to numbers and then begins counting. Over the period of time and with regular practice the little ones develop the understanding of numbers, quantity and the principles of counting.

Natural mathematical thought process is also used by the tiny buddies when they share some of their favourite eatables like chocolates, sweets and candies with their siblings. They raise their hands and ask for 'more' or additional share. They generally compare objects/ items based on the size, shape, more, less, quantity etc.

6.2 WHAT IS NUMBER SENSE?

It is difficult to define precisely, but broadly speaking, it refers to "a well organised conceptual framework of number information that enables a person to understand numbers and number relationships and to solve mathematical problems that are not bound by traditional algorithms" (Bobis, 1996).

Number sense includes a child's understanding towards numbers and quantities viz:

- ✓ Number naming
- ✓ Quantification
- ✓ Counting the number with given quantity
- ✓ Relating numbers with each other
- ✓ Joining numbers (addition)

- ✓ Removing numbers (subtraction)
- ✓ Using math operations and symbols (+, -, ÷, x, =)
- ✓ Employing mental math

6.3 STRATEGIES FOR DEVELOPING NUMBER SENSE

Number sense begins at an early age. Developing a robust number sense during the early years acts as a foundation for learning arithmetic in their coming grades, as it associates counting to quantities. It aids in strengthening and refining the understanding of more and less, helps kids in computational estimation of quantities, recognizing relationship between parts and whole, problem solving and measurements. The following are some of the strategies to develop number sense:

1. Incorporating numbers in day-to-day activities
2. Developing sense of quantity
3. Developing sense of order and relationship with the numbers
4. Splitting numbers and fluency with number operations
5. Understanding zero as absence
6. Sensing of numbers before the written notation for numbers
7. Counting all, counting on (addition & subtraction), counting up(subtraction)
8. Using visualization to solve word problems (addition and subtraction)

Explore games that can assist development of early number sense and what strategies are used for the same.

6.3.1 Incorporating numbers in day-to-day activities




















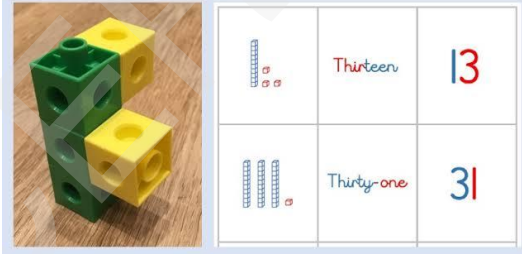
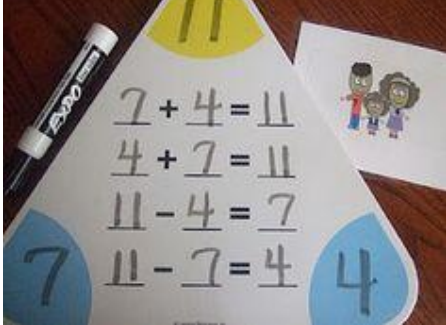
As a teacher you can use the numbers in daily activities that enable the child to do along with you. Let us take few examples like counting same coloured water bottles, pencils of same size, books, and blocks in the classroom. Also, the child may be encouraged to count peanuts or fruits and thus enabling in connecting numbers with respective quantities.



6.3.2 Developing sense of quantity

Children learn the concepts quickly and grasp easily. The best technique for introducing numbers must be done in a fun way that will help them to understand the concepts easily. To develop a sense of quantity flash cards, worksheets may be used as shown in the figs. 1, 2, 3 and 4.

Initially we tend to notice that children memorize numbers, gradually they start to develop their understanding of what a number means and how it can be related to quantity. Then they transfer their learning into daily activities. In this phase of child's journey as a teacher we need to understand that every child is unique and each child learns at his or her own pace considering the individual differences.

	<p>Ring 10 items, and write the numbers.</p> <table border="1"> <tbody> <tr> <td> $12 = 10 + 2$  </td> <td> $16 = \square + \square$  </td> </tr> <tr> <td> $19 = \square + \square$  </td> <td> $17 = \square + \square$  </td> </tr> <tr> <td> $11 = \square + \square$  </td> <td> $20 = \square + \square$  </td> </tr> </tbody> </table>	$12 = 10 + 2$ 	$16 = \square + \square$ 	$19 = \square + \square$ 	$17 = \square + \square$ 	$11 = \square + \square$ 	$20 = \square + \square$ 
$12 = 10 + 2$ 	$16 = \square + \square$ 						
$19 = \square + \square$ 	$17 = \square + \square$ 						
$11 = \square + \square$ 	$20 = \square + \square$ 						
<p>Fig.1 Recognising number with quantity</p>	<p>Fig.2 Developing the sense of quantity 12 as 2 more than 10, 16 as 10+6 or 5+11</p>						
							
<p>Fig. 3</p>	<p>Fig.4 Understanding 11 as sum of 7 and 4 or either</p>						

Relationship between part and a whole	way round. Also 7 as difference of 11 and 4.
---------------------------------------	--

6.3.3 Developing sense of order and relationship with the numbers

Use a blackboard or white board to create a table as shown in Fig.5 and the children may be asked to fill in the missing boxes so that they understand the right ordering of numbers. Similarly, as shown in Fig.6 a shape with numbered dots may be done and a child may be asked to connect the dots or connect-dots books can be used for learning order and relationship with the numbers. Once the shape is completely drawn by the children, they may be asked to colour so as to make learning a bit more fun loving.

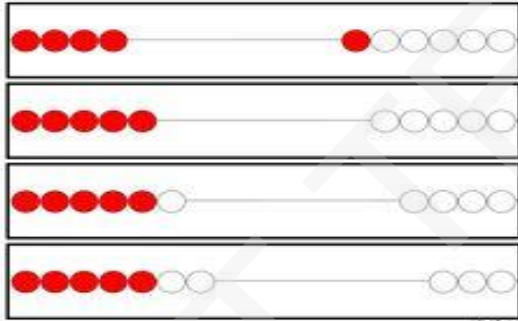

<p>Fig. 5</p> <p>Learning to count and arranging in order</p>	<p>Fig. 6</p> <p>Connecting the dots to understand the ordering of numbers</p>

<p>Fig.7</p> <p>Following an order for jumping and playing the game</p>	<p>Fig.8</p> <p>Using numberline for understanding equidistance, farer and closer numbers</p>
---	---

As the children are familiar with numbers, they may be introduced to number line. Activities can be planned to show distance, equidistance as shown in Fig.7 & Fig.8. Child standing on Number 3 can make out that Number 1 is closer than number 10. Similarly number 15 is at equidistance from number 10 and number 20.

6.3.4 Splitting numbers and fluency with number operations

Children should be taught number bonds as this is the basic foundation for understanding numbers and basic arithmetic. This will in turn assist them to split numbers in different ways. The number may be broken down into component parts or into pairs and at the same time joining back fosters the strategy for number operations and mental problem solving. The figures(9 to 12) given below present an idea about splitting numbers and creating number bonds.

	
<p>Fig.9</p> <p>Basic beads used for splitting numbers</p>	<p>Fig.10</p> <p>Different ways to write a number (splitting and number bonding)</p>

<p>Fig. 11</p> <p>Splitting and joining using basic operation '+'</p>	<p>Fig.12</p> <p>Activity based way to split a number into pairs</p>

6.3.5 Understanding zero as absence

Most little ones can count from one to nine and can match a flash card with quantity. But when it comes to 'Zero' they do not understand the concept from a right standpoint. It is observed that children generally learn the symbol '0' much before they understand it as a mathematical concept. Concept of zero may introduced to them as absence of value. It can be taught through rhymes and planned activities. Some of the learning activities are shown in the figures (13 to 17).

<p>Fig. 13</p> <p>Understanding 'Zero' through rhyme</p>	<p>Fig. 14</p> <p>Circle the tree that has no fruits</p>	<p>Fig.15</p> <p>Zero having no value</p>	<p>Fig.16</p> <p>Addition with zero</p>	<p>Fig.17</p> <p>Subtraction with zero</p>

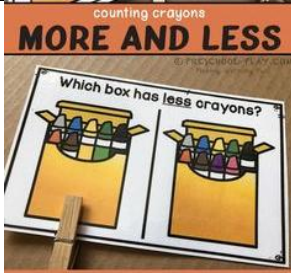


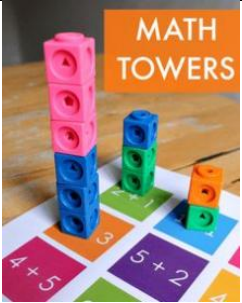
Games can also be planned to teach the concept of zero.

As a teacher you can start with a game where the child begins at the top of the staircase. As and when you call out a number say 'one' the child must take a step down, say you call out number 'two' the child has to take two steps down. If you call out 'zero' the child needs to remain in the same place i.e. top of

the staircase. If the child steps down when you counted ‘zero’ then the child is out of the game. This way the child understands that counting to begin with number that is not ‘zero’.

6.3.6 Sense of numbers before the written notation for numbers

We often see that children sense the concept of numbers well before they start writing and associating with notation of numbers. Children usually repeat the number names in steady sequence, also use one-to-one correspondence. They also understand cardinality (multiplicity) of numbers without using number symbols. Though these concepts may be initially tangled for children but with practice activities, they gain an edge over them. Figures below give an idea of the activities that can be planned.

			
<p>Fig.18</p> <p>Sensing more and less quantity</p>	<p>Fig.19</p> <p>One-to-one correspondence</p>	<p>Fig.20</p> <p>Sensing numbers without written form</p>	<p>Fig.21</p> <p>Cardinality of numbers with operations</p>

6.4 COUNTING

Counting is the ability of match set of items with their relative number name or put names to the quantities. Counting generally means to able to count ones, twos, threes, fives, tens, and more. It is an ability to count the exact number, no more and no less, say counting 1,2,3,4 for four objects.



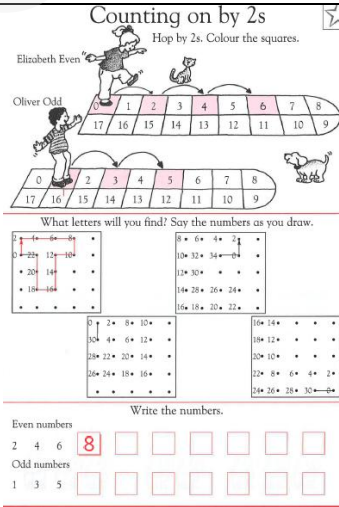
Counting involves how a number system is organized in groups of say 2s or 5s or 10s. We find many children have difficulty with counting skills when it comes to number sequences, skip counting, deriving patterns (odd and even).

Now let us see counting from different perspectives:

- ✓ Counting number combinations
- ✓ Counting all and counting on
- ✓ Counting forward and backward
- ✓ Counting on addition and subtraction without place value
- ✓ Counting up subtraction

6.4.1 Counting number combinations:

There are several activities / games / worksheets that can be planned so that the child is able to count situations not by identifying before and after numbers in isolation but by learning by-heart number combinations.

		 <p style="text-align: center;">Counting on by 2s Hop by 2s. Colour the squares.</p> <p>Elizabeth Even</p> <p>Oliver Odd</p> <p>What letters will you find? Say the numbers as you draw.</p> <p>Write the numbers.</p> <p>Even numbers 2 4 6 8 <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/></p> <p>Odd numbers 1 3 5 <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/></p>
<p>Fig.22</p> <p>Game involving counting by identifying before and after numbers</p>	<p>Fig.23</p> <p>Game involving skip-counting</p>	<p>Fig.24</p> <p>Worksheet for counting by 2s</p>

6.4.2 Counting all and Counting on

‘Counting all’ is simply to begin counting. Here, the kids begin count with the first addend and count the second addend and then count both the addends together beginning at 1. This is typically the first counting strategy for addition that children start with. This is not an effective strategy.

‘Counting on’ is a simple start of mental math approach for addition. ‘Counting on’ means that we begin with an addend and start counting up from there.

Let us say add $4+3$. We say keep '4' (bigger number) in your mind and start counting up '5,6 and 7'. This way we are avoiding them to count 1, 2, 3, 4, 5, 6,7.... It is important that we also teach them the reverse way of counting (commutative property) with respect to addition i.e. " $3+4$ " and ensure that child keeps bigger number in the mind again to begin counting. Though this method can be used for smaller numbers, the same may not be used for bigger numbers say $13+11$. Other advanced addition methods should be used for counting as this is not going to be effective.



Fig.25
Chart showing counting on technique

6.4.3 Counting forward and backward

Learning to count forward and backward is important as this lays the foundation for addition and subtraction.

Draw pathways by writing the missing numbers.

Count back to find out on which step the frog stops.

Write the missing numbers in the boxes.

$3 - 3 = 0$	$20 - 10 =$	$9 - = 6$	$15 - = 5$
$5 - 4 =$	$8 - 8 =$	$5 - = 0$	$20 - = 4$
$15 - 4 =$	$19 - 9 =$	$6 - = 2$	$18 - = 11$
$10 - 9 =$	$16 - 9 =$	$10 - = 4$	$13 - = 10$








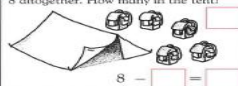


Fig.26
Counting forward and backward

Fig.27
Counting backwards

6.4.4 Counting on addition and subtraction without place value

Using dot patterns, circles representations we can encourage the children to begin with the big number (greater addend) and then count on the dots or circles.

Similarly to solve subtraction number combinations, we tell the children to count backwards. Say For $8 - 3 = _$, students start with 8 and count back 3, “7, 6, 5”. Initially counting down or backwards may be difficult for kids, particularly children with mathematics difficulties as their confidence with counting backwards is less compared to their ease of forward counting. Over a period of time the child realises that subtraction is a mental activity where a procedure required is to count backwards.

<p>Write the sum.</p> <p>$12 + \begin{array}{ c } \hline \cdot \\ \hline \end{array} = _$ $18 + \begin{array}{ c } \hline \cdot \\ \hline \end{array} = _$ $55 + \begin{array}{ c } \hline \cdot \\ \hline \end{array} = _$</p> <p>$8 + \begin{array}{ c } \hline \cdot \\ \hline \end{array} = _$ $25 + \begin{array}{ c } \hline \cdot \\ \hline \end{array} = _$ $13 + \begin{array}{ c } \hline \cdot \\ \hline \end{array} = _$</p> <p>$9 + \begin{array}{ c } \hline \cdot \\ \hline \end{array} = _$ $30 + \begin{array}{ c } \hline \cdot \\ \hline \end{array} = _$ $23 + \begin{array}{ c } \hline \cdot \\ \hline \end{array} = _$</p> <p>Use the ten frames to solve each equation.</p> <p>$7 + 4 = _$  $12 + 2 = _$ </p> <p>$9 + 3 = _$  $13 + 4 = _$ </p>	<p>Say and count as you write.</p> <p>10 altogether. How many in the tent?  $10 - 6 = 4$</p> <p>18 altogether. How many in the tent?  $18 - 8 = 10$</p> <p>9 altogether. How many in the tent?  $9 - _ = _$</p> <p>8 altogether. How many in the tent?  $8 - _ = _$</p> <p>Say as you write.</p> <p>$16 - 4 = 12$ $18 - _ = 7$ $12 - _ = 2$</p> <p>$15 - _ = 14$ $19 - _ = 5$ $15 - _ = 9$</p> <p>$9 - _ = 4$ $17 - _ = 11$ $11 - _ = 10$</p> <p>Say as you write.</p> <p>$15 - 5 = 10$ $10 - _ = 0$ $16 - 0 = _$</p> <p>$13 - 10 = _$ $20 - _ = 0$ $8 - 8 = _$</p>
<p>Fig.28</p> <p>Counting on – addition</p>	<p>Fig.29</p> <p>Counting on – subtraction</p>
	
<p>Fig.30</p> <p>Counting on with legos</p>	<p>Fig.31</p> <p>Counting on – subtraction with pebbles</p>

6.4.5 Counting up subtraction strategy

Counting up method is a method of subtraction where the focus is on addition. This is alternative method from the usual method of subtraction.

We first think from an angle of addition here. For example $10 - 8 = ?$ The child begins to think from the angle of addition as to how much needs to be added to number 8 to reach number 10.

$$8 + \underline{\quad} = 10 \text{ and arrive at } 2.$$

Count Up to Subtract	
<small>Directions: Solve the first subtraction equation. Use the second equation to help you. Think of it as an addition equation. Count up from one addend to find the unknown addend.</small>	
1. $10 - 8 = 2$ find it this way... $8 + ? = 10$	2. $20 - 5 = 15$ find it this way... $5 + \underline{\quad} = 20$
3. $16 - 7 = \underline{\quad}$ find it this way... $7 + \underline{\quad} = 16$	4. $18 - 9 = \underline{\quad}$ find it this way... $9 + \underline{\quad} = 18$
5. $12 - 10 = \underline{\quad}$ find it this way... $10 + \underline{\quad} = 12$	6. $19 - 9 = \underline{\quad}$ find it this way... $9 + \underline{\quad} = 19$
7. $17 - 5 = \underline{\quad}$ find it this way... $5 + \underline{\quad} = 17$	8. $11 - 6 = \underline{\quad}$ find it this way... $6 + \underline{\quad} = 11$
9. $14 - 6 = \underline{\quad}$ find it this way... $6 + \underline{\quad} = 14$	10. $10 - 4 = \underline{\quad}$ find it this way... $4 + \underline{\quad} = 10$
11. $13 - 8 = \underline{\quad}$ find it this way... $8 + \underline{\quad} = 13$	12. $15 - 2 = \underline{\quad}$ find it this way... $2 + \underline{\quad} = 15$

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Fig. 32

Counting up subtraction strategy

6.5 USING VISUALISATION TO SOLVE WORD PROBLEMS (ADDITION AND SUBTRACTION)

Visualisation is an ability to see and understand the current context of the situation in order to solve the problem. It opens the door for greater understanding, tapping on the memory and recall and moves towards problem solving. This is one of the most important process in learning and is a critical component of problem representation.

You as a teacher play a significant role in teaching children how to visualize the mathematical problems using pictures, diagrams, graphical displays, tables, models or may be by your drawing. Creating these mental images is very crucial, as the representations should be able to show the relationships with the problem parts.

Word problems play a major role in making math meaningful and contextual for kids as they associate reasoning in classroom context. Everyday problems and situations can be put in simple way so that children find it easy to visualize the context of word problem. The children may be given word problems as stories in order to draw their attention and interest.

Word problems become simple if children

- Develop the habit of visualizing
- Identify words that are likely to be used in the problem (using ‘and’ for ‘+’, ‘how many’, ‘totally’, ‘altogether’ for addition and using ‘how many more’ for subtraction)
- Group problems based on type
- Interpret what is happening in the problem

Here are some examples of word problems:

1. Raj has 4 pink balloons, Lilly has 5 red balloons. How many balloons are there altogether now?
2. 3 children are there in the swimming pool. 4 more jumped into the pool. Totally how many children are there in the pool?
3. Ashu has 3 red books and 6 yellow books. How many books are there with Ashu?
4. You found four apples. Your friend found 7 apples. How many more apples did your friend find?
5. Manas has 8 chocolates and Nikki has 5 chocolates. Who has more chocolates and how many more?

What are the simple steps for solving word problems?
--

6.6 IMPORTANCE OF OBSERVATION STRATEGIES

Observational strategies lay the foundation for mathematical skills in early years of childhood. In order to add and subtract correctly the kids need to sharpen their observation.

For a given problem to be solved the child needs to correctly read the problem in its entirety. The next major step involves observation where the child needs to understand the problem, decides on the most significant aspects and insignificant aspects of the given problem. This is very important and difficult step for most young learners. Once this observation strategy is mastered, then problem solving is just simple. As a teacher we know that classroom is filled with opportunities to learn and practice, hence several activities can be planned in the setup.

Observation involves the following:

- One-to-one correspondence between numbers / symbols / items / objects
- Variation in size, shape or pattern
- Associating with daily routines or everyday life

Why mathematical skills need to be developed at an early age?

6.7 RECOGNISING MULTIPLE WAYS OF ADDITION AND SUBTRACTION

There are various ways of recognizing addition and subtraction. Some of them are listed below:

6.7.1 Based on terminology:

Addition: Using the keywords 'plus, add, sum, put together, altogether, join, combine, in all, total' etc.

Subtraction: Using the keywords 'minus, take away, difference, compare, separate, how much more, remaining, how many left' etc.

6.7.2 Based on a specific strategy:

Generally during the number talk, children are given few problems related to a specific strategy that involve combining or separating. Here, the child needs to begin with unknown which is quite challenging.

Combining

Result unknown

There are five sheep in a farm.
Three more sheep joined into the farm.
How many sheep are in the farm? $5 + 3 = \underline{\quad}$

Change unknown

There are five sheep in the farm.
Some more sheep walked into the farm.
There are now eight sheep in the farm.
How many sheep came into the farm? $5 + \underline{\quad} = 8$

Start unknown

There were some sheep in the farm.
Three sheep came into the farm.
There are now eight sheep in the farm.
How many sheep were in the farm to start with? $\underline{\quad} + 3 = 8$

Separating

Result unknown

There were eight sheep in the farm.
Five sheep ran out of the farm.
How many sheep are still in the farm? $8 - 5 = \underline{\quad}$

Change unknown

There were eight sheep in the farm.

Some ran away.
Three sheep are still in the farm.
How many sheep ran away? $8 - \underline{\quad} = 3$

Start unknown There were some sheep in the farm.
Five sheep ran away.
Now there are three sheep in the farm.
How many sheep were in the farm to start with? $\underline{\quad} - 5 = 3$

6.7.3 Based on parts and whole:

The problem needs to be understood in its totality, and the same should be broken down into smaller parts (significant aspects to be separated and irrelevant aspects to be eliminated) for further understanding.

Explore some more ways for recognising addition and subtraction in problem solving.

6.8 SUM UP:

During the early years skills related to number sense act as a strong indicator for overall math achievement in children. A robust groundwork in number sense also prepares the children to be flexible in their problem solving. During this process of learning we tend to understand that children learn numbers as being meaningful and regardless of manipulation, their outcomes are unvarying and sensible.

We have seen several strategies of developing number sense. As an early childhood educator you have a big responsibility in laying a strong foundation for building mathematical knowledge and skills among children for their future learning in math.

6.9 QUESTIONS:

1. As an early childhood educator, why do you think developing number sense is important for children?
2. Explain counting all, counting on, counting backward and counting up with examples.
3. How to solve subtraction word problems using visualization? Give 2 examples.
4. State 5 important ways to teach children the concept of addition.

UNIT 7 - DEVELOPING NUMBER SENSE UP TO 100

7.0 OBJECTIVES

After completing this unit as a preschool teacher trainee, you will be able:

- To understand the importance of number sense and strategies used for developing number sense up to 100
- To demonstrate the counting techniques in classroom activities
- To develop sense of numbers and counting numbers up to 100, counting in bundles of 10, counting forward and backward by 10s
- To explain place value concept
- To design worksheets to do addition and subtraction in different ways
- To analyse the common errors that children make in addition and subtraction

7.1 INTRODUCTION

Number sense denotes giving meaning to the numbers, how these numbers are related to each other and their relative magnitude. We have learnt number sense in our previous classes. We will extend our knowledge on how we can develop number sense beyond 20 in our students. In this unit we will be learning on how to develop number sense up to 100 for little older children in the age group 4 or 5.

Learning about number sense is important as numbers play a vital role in our day to day activities. Exploring and playing with numbers is a lifelong activity. As teachers we need to help our students in exploring numbers and they should be encouraged to think on various relationships that can be formed with numbers to arrive at patterns.

7.2 COUNTING NUMBERS

Counting numbers up to 100 is a fundamental part of number sense. Here are some activities that can be tried to help children in counting.

7.2.1 SENSING NUMBERS AND COUNTING NUMBERS UP TO 100

Firstly it is important that children learn numbers beyond 20. As a teacher you need to bring in the understanding that there are numbers beyond 20, for this thought, activities are to be planned. The following figures assist you in your planning of activities.



Fig. 1

First begin teaching up to 50 and then extend till number 100. The children must be taught both in numbers and figures.

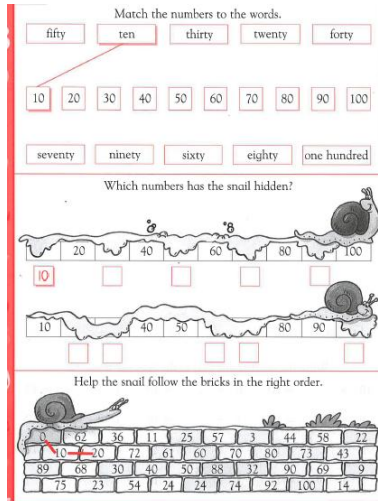


Fig:2

Activities to count numbers in 10 and placing numbers in right order should be practiced



Fig.3

Counting pebbles/ ice-cream sticks up to 100 can be encouraged

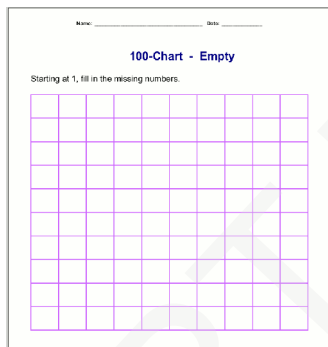


Fig.4

Empty 100 chart to be given for practice to the students

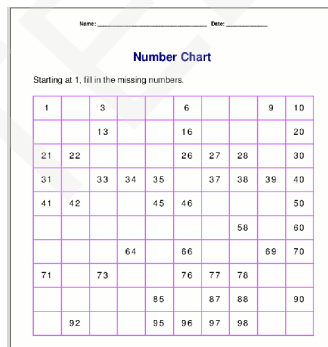


Fig.5

Missing numbers up to 100 chart to be given for the students

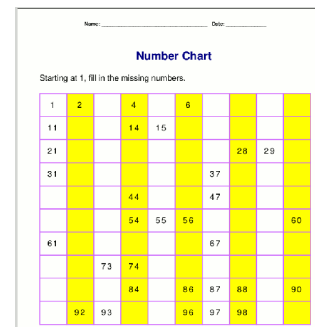


Fig.6

Missing numbers up to 100 chart to be given for the students

7.2.2 COUNTING IN THE BUNDLES OF 10 AND COUNTING FORWARD & BACKWARD BY 10s BY 10s

Counting in the bundles of tens is a crucial skill that constructs number sense. It promotes the taught about counting larger numbers and enables to think about different number structures. This conceptual understanding of round numbers is essential for the students. Counting on forward and backward by 10s is also equally important.



Fig.7

Counting in bundles of 10

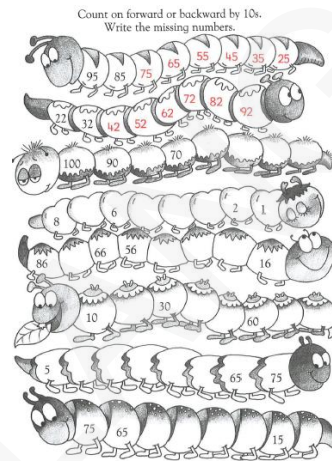


Fig.8

Caterpillars worksheet - Counting forward and backward by 10s with missing numbers

7.3 CONCEPT DEVELOPMENT FOR ORDER AND RELATIONSHIP WITH NUMBERS

For developing order and relationship with numbers, we as teachers need to extend the understanding of number line. This is an effective way to increase the child's experience beyond number 20.

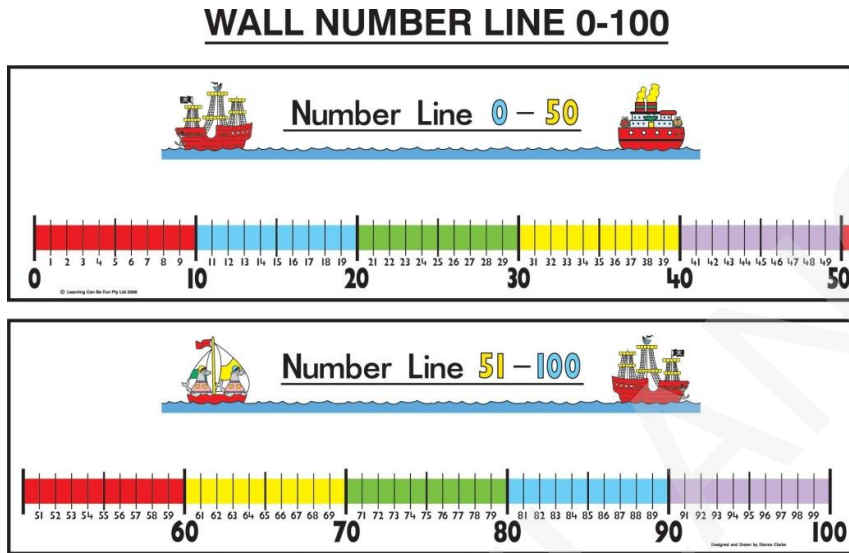


Fig.9

Showing number line from 0 to 100

Begin with teaching numbers in families. Let us talk about 30s family and 60s family here:

30s family

<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

30	32	34
31	35	33
37	38	39
36		

60s family

<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

62	60	61
68	63	66
64	67	65
69		

7.4 CONCEPT DEVELOPMENT FOR ARRANGING NUMBERS IN ASCENDING AND DESCENDING ORDER

Children may be given different number families and they may be asked to arrange the same in a sequential order (either ascending or descending order)

Practice may initially begin for ascending and descending order by taking only 3 different numbers and may be extended to more numbers.

Start taking any 3 numbers and the child may be asked to arrange in ascending order



The child knows there are 6 ones in 16, 1 one in 11 and 4 ones in 14 as tens place is same.

According to ascending order, the smallest should come at the beginning and the greatest number at the end.

So the child will be able to write them in ascending order by interchanging the bricks given above.



Similarly practice may be given for more numbers, different numbers in 10s and ones places.

Start taking any 3 numbers and the child may be asked to arrange in descending order



The child knows there are 3 ones in 23, 5 ones in 25 and 1 one in 21 as tens place of 2 remains is same.

According to descending order, the greatest should come at the beginning and the smallest number at the end.

So the child will be able to write them in ascending order by interchanging the bricks given above.



Similarly practice may be given for more numbers, different numbers in 10s and ones places.

Also, the practice with different number families can be carried out so that the child masters arrangements in different forms.

7.5 DEVELOPING PLACE VALUE CONCEPT

We must have observed that when kids begin counting, they just mostly learn numbers as a continuation that keeps going on and on without actually understanding the basic structure of our numerical system. They need to develop understanding of grouping in the tens, hundreds and so on. This is not possible for small kids, the child around the age group of 4+ years will be able to typically understand the place value. So, let us bring in this concept for a little older child by doing some activity.

Activity: Pile several tamarind seeds (or many erasers or coins) on a work table and demonstrate how to count these in a group of ten. Initially make a group of ten and similarly go on to make some more ten groups and keep the left over seeds separately.

Similarly do continue in the same way with coins and erasers by counting in tens and keeping the left over ones separately. In this process introduce the words twenty, thirty, forty, fifty... etc. In this way the child will be able to understand the tens place and the left over coins / erasers in ones place. The tens place may have not just ten, it could be twenty, thirty, forty or fifty. But the left over items are likely to be below number nine as they cannot be put in the bundle of Ten.

The following figures provide various ways to introduce place value fundamentals into a child.

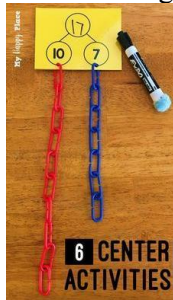


Fig.10
Place value of tens and ones for number 17. Also showing the same with clips

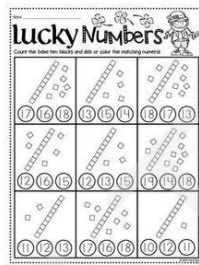


Fig.11
Lucky number to be picked based on the blocks



Fig.12
Place value for numbers below 20



Fig.13
Place value for numbers greater than 20

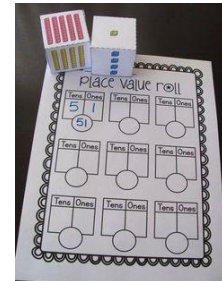


Fig.14
Worksheet based on place value roll or instead use dice

As a teacher, you need to provide illustrations based on the overall perspective based on the above concepts of number line, order & sequence; and place value.

For example:

Consider a number 42.

- A child from **number line** understands that 42 falls between 40 and 50.
- The child gets the clarity that 42 is closer to 40 based on **place value** of 40 and place value of 50.

- Taking into consideration 42, 40 and 50, the child can arrange in ascending order as 40, 42 and 50 based on order and sequence.

Explore more ways to give overall perspective based on the concepts of number line, order & sequence; and place value.

7.6 DIFFERENT WAYS TO DO ADDITION BASED ON NUMBER SENSE

Children should be taught to add and subtract using different strategies as these form the fundamental operations for their overall computational learning. Here are some strategies of doing addition along with illustrations. You can develop worksheets or activities based on these strategies.

S.No.	Strategy	Illustrations
-------	----------	---------------

1. **Addition of single digit addends**

Ex.1 Ex.2

$$\begin{array}{r} 5 \\ + 8 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ + 4 \\ \hline \end{array}$$



2. **Addition of many single digit addends**



Ex.1 Ex.2

$$\begin{array}{r} 4 \\ 8 \\ 1 \\ 6 \\ + 7 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ 4 \\ 2 \\ 3 \\ + 5 \\ \hline \end{array}$$





3. **Addition in columns with 2-digit 2 addends**

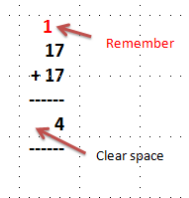
Doing addition by writing one number below the other number and then *adding one column at a time*.

Ex.1	Ex.2
1 5	2 4
+ 2 2	+ 3 2
	

4. **Addition in columns with carry forward**

Doing addition by writing one number below the other number and carrying the sum of ones column to the tens column.

Ex.1	Ex.2
1 7	4 7
+ 1 7	+ 3 9
	



When we add 7 and 7 in the ones column we get 14.

Child understands that 14 can be written as 1 Ten and 4 ones. So take the 1 Ten to the top of tens column and write the 4 in the ones place.

While adding tens column remember to add that 1 ten to the sum of tens column.


Similarly knowledge can be extended for bigger numbers and 3-digit numbers too.


5. **Addition in columns with 1 or 2 digit numbers with carry**

Ex.1 Ex.2


$$\begin{array}{r} 19 \\ + 6 \\ \hline \end{array}$$
$$\begin{array}{r} 7 \\ + 65 \\ \hline \end{array}$$


6. **Horizontal Addition of 2-single digit addends**

Ex.1 $6+3=$ 


Ex.2 $7+9=$ 


7. **Horizontal addition of 2-double digit addends**

Ex.1 $11+14=$ 


Ex.2 $33+22=$ 


8. **Horizontal addition of 2-double digit addends with carry forward**

Ex.1 $29+64=$ 


Ex.2 $28+52=$ 


9. **Horizontal addition with 1 or 2 digit addends**


Ex.1 $7+85=$ 


Ex.2 $93+6=$ 

10 **Horizontal addition with ending in fixed number**

Ex.1 $10+20=$ 

Ex.2 $30+50=$ 

Ex.3 $45+25=$ 

Ex.3 $25+65=$ 

11 **Word problems on addition**

Ex.1

In Anitha's house there are 12 big trees and in Mohan's house there are 21 small trees. How many trees are there totally?

Ex.2

Megha has 35 birds in her farm. Joseph's farm is next to Megha's farm. Joseph has 57 birds with him. Altogether how many birds are there?

Ex.3

Eshu has 36 pebbles and Mani clubbed his 27 pebbles along with Eshu in a box. Combined together how many pebbles are there in the box?

7.7 DIFFERENT WAYS TO DO SUBTRACTION BASED ON NUMBER SENSE

Subtraction is equally important for children. Here are some strategies of subtraction that as a teacher you should ensure perfection with your classroom kids. You can develop worksheets or activities based on these strategies.

S.No.	Strategy	Illustrations
-------	----------	---------------

1. **Subtraction of single digit numbers**

Ex.1 Ex.2

$$\begin{array}{r} 9 \\ - 3 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ - 5 \\ \hline \end{array}$$



2. **Subtraction in columns with 2-digit numbers**

Ex.1 Ex.2



$$\begin{array}{r} 35 \\ - 22 \\ \hline \end{array} \quad \begin{array}{r} 84 \\ - 51 \\ \hline \end{array}$$

Doing Subtraction by writing one number below the other number and then *subtracting one column at a time.*



3. **Subtraction in columns with borrow**



Doing Subtraction by writing one number below the other number. As the right side top number is smaller than the right side below number, we cannot subtract.

Ex.1	Ex.2
$\begin{array}{r} 36 \\ - 29 \\ \hline \end{array}$	$\begin{array}{r} 87 \\ - 59 \\ \hline \end{array}$
	


So we need to subtract 1 from the top number in the column directly to the left of the number, then cross it out because you are borrowing from it. Subtract 1 and write the answer above the number you have crossed out.


Now add 10 to the top number on the right in the column that you are working on. As the number now gets bigger than the below number, you can proceed with your subtraction.

4. **Subtraction in columns with 1 or 2 digit numbers**


Ex.1	Ex.2
$\begin{array}{r} 38 \\ + 9 \\ \hline \end{array}$	$\begin{array}{r} 47 \\ + 7 \\ \hline \end{array}$
	


5. **Horizontal Subtraction of 2-single digit numbers**

Ex.1 $6-4=$ 


Ex.2 $7-2=$ 

6. **Horizontal Subtraction of 2-double digit numbers**

Ex.1 $19-11=$ 

Ex.2 $33-23=$ 


7. **Horizontal Subtraction of 2-double digit numbers with borrowing**

Ex.1 $97-69=$ 


Ex.2 $58-47=$ 


8. **Horizontal Subtraction with 1 digit numbers**


Ex.1 $84-9=$ 


Ex.2 $93-6=$ 

9. **Horizontal Subtraction with ending in fixed number**

Ex.1 $90-40=$ 

Ex.2 $33-13=$ 

Ex.3 $65-25=$ 

Ex.4 $91-61=$ 

10. **Word problems on subtraction**

Ex.1

Moni has 38 seeds and from that she gave 19 seeds to her sister. How many seeds are left with Moni now?

Ex.2

Lalli is having 71 biscuits. Lalli ate 34 biscuits in a week. How many remaining with her?

Ex.3

88 colour pencils are there with Roja. 51 colour pencils are there with Charan. Who has more pencils and how much more?

7.8 DIFFERENT WAYS OF ADDITION & SUBTRACTION BASED ON NUMBER SENSE

There are different ways in which a child should be taught to add and subtract simultaneously. Though this concept is a bit tough for the child to catch up, with regular practice and drill this can be mastered. Here are some strategies where addition and subtraction are used together.

S.No.	Strategy	Illustrations
-------	----------	---------------

1. **Missing addends**

Ex.1 $37 + \underline{\quad} = 39$

Ex.2 $\underline{\quad} + 5 = 40$

Ex.3 $65 = 45 + \underline{\quad}$

Ex.4 $81 = \underline{\quad} + 36$

2. **Word problems based on addition and subtraction together**

Ex.1

Padma has 47 flowers. Mini has 50 flowers. Padma gave 20 flowers to Ganga and Mini gave 27 flowers to Ganga. How many flowers are there with each of them now?

Find out other strategies where both addition and subtraction can be used together.

7.9 COMMON ERRORS DONE BY CHILDREN IN ADDITION & SUBTRACTION

We notice that children make many mistakes while they are doing addition and subtraction. Here are some of the errors that we generally tend to see:

S.No.	Addition Problem	Solution given by Children	Error	Suggestion to the teacher
-------	------------------	----------------------------	-------	---------------------------

1. Add the following numbers

$$\begin{array}{r} 15 \\ + 27 \\ \hline \end{array}$$

$$\begin{array}{r} 15 \\ + 27 \\ \hline \end{array}$$



Carry forward not used

Before starting addition give sufficient practice for the place value.

Show the example with some beads chain.

2. Add the following numbers

$$\begin{array}{r} 45 \\ + 25 \\ \hline \end{array}$$

$$\begin{array}{r} 45 \\ + 25 \\ \hline \end{array}$$



Addition is done from left hand side

Explain that addition should begin from right side numbers and give sufficient practice.

Use some match sticks to explain.

3. Add the following numbers

$$\begin{array}{r} 48 \\ + 5 \\ \hline \end{array}$$

$$\begin{array}{r} 48 \\ + 5 \\ \hline \end{array}$$



Wrong arrangement of numbers in the columns

Concept of place value to be strengthened by the teacher.

4. Add the following numbers

$$\begin{array}{r} 44 \\ + 59 \\ \hline \end{array}$$

$\begin{array}{r} 3 \\ 44 \\ + 59 \\ \hline 1 \end{array}$

Addition carry forward from left hand side

Teach the concept through expanded form

4 tens + 4 ones to be added to 5 tens and 9ones.

5. Add the following numbers

$$\begin{array}{r} 76 \\ + 19 \\ \hline \end{array}$$

$\begin{array}{r} 76 \\ + 19 \\ \hline 85 \end{array}$

Addition carry forward forgot to carry

Concept of addition is clear but needs to be strengthened by practice as it's a careless mistake

6. Add the following numbers

$$\begin{array}{r} 23 \\ + 34 \\ \hline \end{array}$$

$\begin{array}{r} 23 \\ + 43 \\ \hline 66 \end{array}$

Reverse way to write numbers in columns

Reinforce on observation skills

S.No.	Subtraction Problem	Solution given by Children	Error	Suggestion to the teacher
1.	Subtract the following numbers 26 and 29	$\begin{array}{r} 26 \\ - 29 \\ \hline 3 \end{array}$	Subtracted 6 from 9 which is not possible. Haphazard way of writing in the columns.	Concept of larger and smaller number to be made clear.
2.	Subtract the following numbers 100 and 58	$\begin{array}{r} 100 \\ + 58 \\ \hline 58 \end{array}$	Subtraction is done as 8-0 and 5-0 instead of borrowing	Subtraction worksheets with borrowing need to be given more practice. Also subtraction with '0' should be made clear.
3.	Subtract the following numbers 33 and 21	$\begin{array}{r} 33 \\ - 21 \\ \hline 54 \end{array}$	Arrangement is not rightly done but instead of subtraction, the child does addition.	Importance of sign (- minus) should be stressed
4.	Subtract the following numbers	$\begin{array}{r} 54 \\ - 29 \\ \hline 35 \end{array}$	Borrowing done properly but a ten to be reduced from 5 on left hand side is not taken into account.	After strikethrough the child should be made to write a number less by 1 need to be insisted.

We also find kids making errors in word problems related to addition and subtraction.

For example the child understands whether the problem is related to addition or subtraction but is unable to write the correct statements.

S.No.	Subtraction Problem	Solution given by Children	Error	Suggestion to the teacher
1.	Subtract the following numbers 26 and 29	$\begin{array}{r} 26 \\ - 29 \\ \hline 3 \end{array}$	Subtracted 6 from 9 which is not possible.	Concept of larger and smaller number to be made clear.
2.	Subtract the following numbers 100 and 58	$\begin{array}{r} 100 \\ + 58 \\ \hline 58 \end{array}$	Subtraction is done as 8-0 and 5-0 instead of borrowing	Subtraction worksheets with borrowing need to be given more practice. Also subtraction with '0' should be made clear.
3.	Subtract the following numbers 33 and 21	$\begin{array}{r} 33 \\ - 21 \\ \hline 54 \end{array}$	Arrangement is not rightly done but instead of subtraction, the child does addition.	Importance of sign (- minus) should be stressed
4.	Subtract the following numbers	$\begin{array}{r} 54 \\ - 29 \\ \hline 35 \end{array}$	Borrowing done properly but a ten to be reduced from 5 on left hand side is not taken into account.	After strikethrough the child should be made to write a number less by 1 need to be insisted.

Similarly, find some more general mistakes that a child makes in word problems related to addition and subtraction.

7.10 SUM UP:

Number sense is a big bag that needs to be unpacked and taken to our classrooms. We have seen different strategies to develop number sense in the previous chapter and this chapter. As a teacher it is required for you to construct number sense in children by making them understand the concept of number, develop their mental math skills and provide them with tools to look at math beyond the classroom into the outside world and make associations and comparisons. In this process awareness of the relationship with number and its quantity should be developed.

Number sense expands while working on several opportunities and with time. This can be planned by the teacher through play way methods, games, adding them in real world contexts.

7.11 QUESTIONS:

1. Why counting is a fundamental part of developing number sense? Explain with few activities in context of environment / real world applications.
2. How to develop place value concept among children?
3. Explain 3 ways of doing addition and 3 ways of doing subtraction based on number sense.
4. State few common errors that children make while adding numbers.

UNIT 8 - SPACE, SHAPES AND PATTERNS

8.0 OBJECTIVES

After completing this unit as a preschool teacher trainee, you will be able:

- To understand basic shapes
- To explain point, line and different shapes
- To identify shapes
- To compare different shapes
- To distinguish between flat and curved surfaces related to shapes
- To associate different shapes with real world objects
- To develop familiarity with Van Hiele levels of learning geometry
- To develop activities, worksheets to teach shapes, patterns and position of objects

8.1 INTRODUCTION

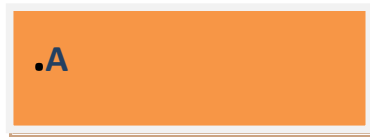
It may be observed that children start to recognize and interrelate with their surroundings from their early years. This lays the foundational learning for spatial relationships. As children grow they improve on this understanding of dimension, space, volume, size, movement and many other factors that establish several inter-relationships between various items, objects and people. As a teacher it is our task to foster the child's ability to understand these special associations from numerous perspectives when the child enters different learning phases.

When the child reaches school, the child is in a position to move around without much help, can interact with others in the social setting, able to walk through the entrances and sit comfortably at his/her table. This kind of movement is all set to support concepts of space, direction, dimension and spatial relationship in the given surroundings.

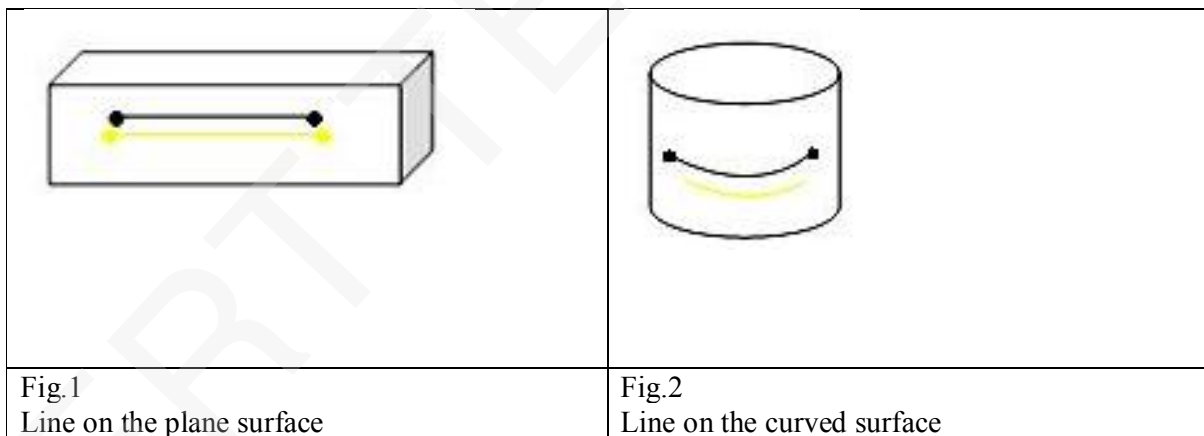
It is required to sharpen and refine these skills through activities (simple to complex) that give confidence and comprehension of spatial relationships. Concepts related to size, shape should be reinforced by giving age-appropriate puzzles, sculpting with clay to get a grasp of interrelated parts and dimensions. Spatial relationships can also improve with activities involving cycling, drawing, jewellery-making with beads, basic carving or carpentry that help in promotion of understanding size and dimension. Now let us see how to introduce points, lines and then move on to shapes in our classroom.

8.2 POINTS & STRAIGHT LINES

The fundamental element of geometry begins with a **Point**. It can be considered as a smallest dot or a very tiny mark on the paper or the smallest dot that we put with tip of a pencil. This negligible mark denotes the existence of a point. A point does not have any length, breadth or thickness, it has a definite position. It is denoted with any capital letter in English say, A, B, C, or P, Q



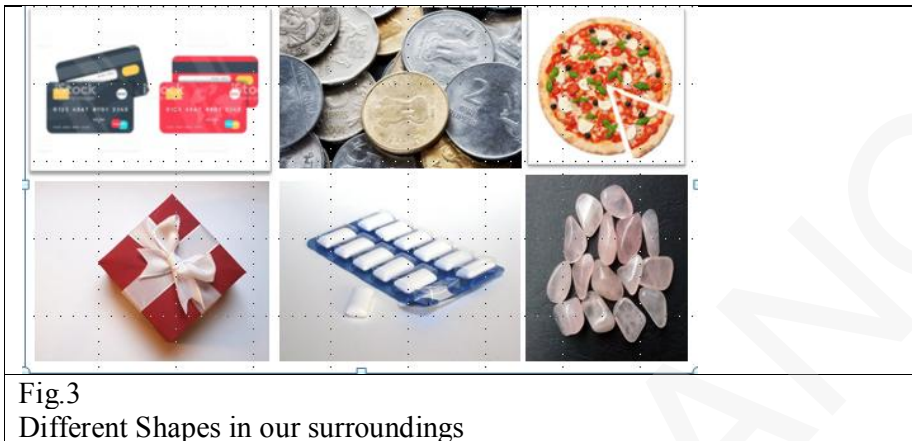
A line is formed when two points at a distance on a surface are joined by pencil. A line can be on a plane surface as well as on the curved surface.



A line can have any orientation or length. *If the line is vertical it's a standing line, if the line is horizontal it's a sleeping line. So a straight line can either be a standing line, or sleeping line or a diagonal line connected by two points.*

8.3 INTRODUCING SHAPES

We consciously and unconsciously take a look at several bits and pieces in this material world that is full of different shapes. Our focus in math generally regulates around two-dimensional objects and three-dimensional solids. The basic two-dimensional shapes are rectangle, square, circle, oval and basic three-dimensional shapes are cylinder, cube, cuboids, prism etc. that we find around us.



Examples of doors, windows, balloons, finger rings, kites, bangles, tall building complexes, cycle wheels, tiffin boxes etc. can also be shared with children.

Shapes can be classified as open and closed shapes.

Open Shape	Closed shape

8.3.1 CONCEPT OF POLYGON

Any flat shape that has three or more straight sides and if it is closed, then it is called as a polygon.

No. of sides	Shape
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon




8.3.2 TYPES OF 2-DIMENSIONAL & 3-DIMENSIONAL CLOSED SHAPES

We will classify 2-Dimensional shapes into:








- Circular shapes
- Polygons






2-Dimensional shapes are have length, width/breadth. They do not have thickness. Here is the list of 2-dimensional closed geometric shapes.

CIRCULAR SHAPES & THEIR PROPERTIES:

Shape	Image	Description & examples(informal words from regular day-to-day vocabulary)
Circle		A circle is round shaped having the same radius from a fixed point in the center. Ex. Cycle wheels, Pizza,
Semi - circle		A circle that is cut exactly in half along its diameter is a semi-circle Ex. half a pizza, half a cookie
Oval		A prolonged circle where the radius is shorter on one axis than the other Ex. eggs, buns for a hot dog, a running track



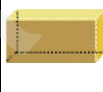
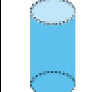
PROPERTIES OF POLYGONS AND THEIR INTERRELATIONSHIPS

Shape	Image	Description & examples(informal words from regular day-to-day vocabulary)
Triangle		Three-sided closed figure joined by straight lines Ex. a diagonally cut sandwich
Square		Four equal straight sides with four right angles joined by four lines. Ex. Tiles of a flooring
Rectangle		Four straight sides with four right angles but having different length and breadth. If the length and breadth are equal then a rectangle becomes a square. Ex. A notebook, mobile phone, slate
Parallelogram		Four-sided figure with two pairs of parallel sides of equal length and equal breadth The opposite angles may not be right angles. If the opposite angles are right angles then it becomes a rectangle. Ex. Some erasers
Rhombus		A parallelogram with equal length sides. The angles may not be right angles. If a rhombus can be a square if it has all four angles as right angles. Ex. Some kites, some crystals
Trapezium		Four-sided figure with just one pair of parallel sides Ex. a popcorn box
Pentagon		Five straight sides of equal length Ex. Shape found in soccer balls

Hexagon		Six straight sides of equal length Ex. hardware nuts, beehive cells, old coins
Heptagon		Seven straight sides of equal length Ex. covers for a cookie box, some types flower vases
Octagon		Eight straight sides of equal length Ex. Some umbrella, road sign boards
Nonagon		Nine straight sides of equal length Ex. Some containers, cookie boxes
Decagon		10 straight sides of equal length Ex. Some coin collection boxes

3-DIMENSIONAL SHAPES & THEIR PROPERTIES

They have length, width and height or thickness. Here is the list of 3-dimensional closed geometric shapes.

Shape	Image	Description & examples
Sphere		A three-dimensional circular shape Ex. Ball, Orange fruit,
Cube		A three-dimensional square box having equal sides Ex. Rubik Cube, Dice, Sugar cube, Ice cube
Cuboid or Rectangular Prism		A three-dimensional rectangular box having opposite sides equal Ex. Cartoon box, Pencil box
Cylinder		A three-dimensional figure with parallel sides and a circular cross-section Ex. Cardboard inside a paper towel, a drinking glass

Explore some shapes both 2D and 3D that we find in our day to day life.

DEFINING AND NON-DEFINING ATTRIBUTES:

Every shape has defining and non-defining attributes. Let us see the differences:

Defining attributes	Non-defining attributes
<ul style="list-style-type: none">• No. of sides could be parallel or non-parallel or straight or curved• No. of vertices or a vertex• No. of edges• No. of faces	<ul style="list-style-type: none">• Colour• Size• Orientation
<i>A shape can change if there is a change in the defining attributes.</i>	<i>A shape will not change if there is a change in non-defining attributes.</i>

8.4 FLAT AND CURVED SURFACES

2D objects have flat surfaces also known as plane surfaces. We find flat surfaces for rectangle, square, triangle etc. The objects whose surfaces are not flat but are rounded are called as curved surfaces. Examples of curved surfaces are soccer ball, Orange fruit. We also have shapes that have both flat and curved surfaces such as cone, cylinder etc.

Rolling and sliding of objects:

It is often noticed that any object placed on a ramp will either roll, or slide or stay intact. This depends of the shape of the object. If the object is having curved surfaces, then it is more likely to roll or slide.

Objects	Items
Roll	Football, Tennis Ball, Lemon, Orange
Slide	Notebook, Duster, Eraser
Both slide and Roll	Pen, Coin, Pencil, Toy wheels

Predict the rolling and sliding behaviour of objects.

8.5 FAMILIARITY WITH THE DISCUSSION ON *VAN HIELE LEVELS* IN LEARNING GEOMETRY

There were two Dutch teachers Hiele-Geldof and Dina who turned into researchers. They found some of their students faced difficulties in learning geometry based on their classroom experiences over the years. In order to deal with these problems they devoted their life time to their theory and came up with three aspects:

- The existence of levels
- The properties of the levels
- The progress from one level to the next level

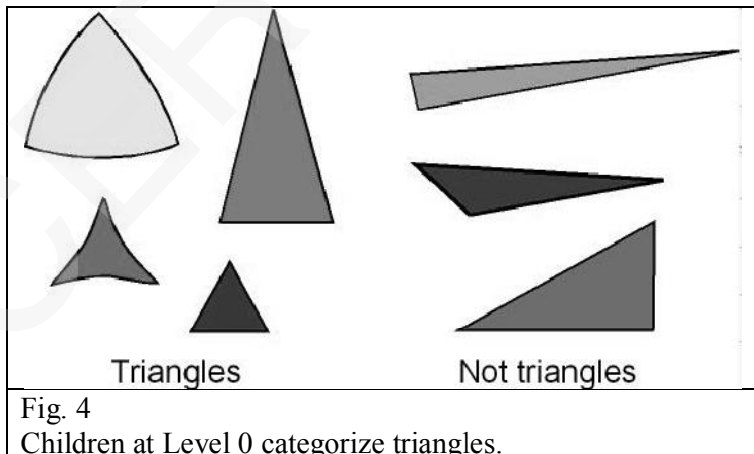
According to their theory, there are five levels of thinking or understanding in geometry:

- Level 0 Visualization
- Level 1 Analysis
- Level 2 Abstraction
- Level 3 Deduction
- Level 4 Rigor

Based on **I.Vojkuvkova** (2012) in his paper “The van Hiele Model of Geometric Thinking” we find the following:

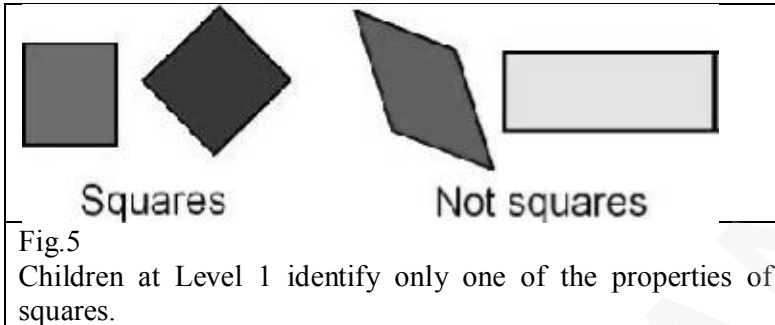
Level 0 Visualization (Basic visualization or Recognition)

At this level pupils use visual perception and nonverbal thinking. They recognize geometric figures by their shape as “a whole” and compare the figures with their prototypes or everyday things (it looks like door”), categorize them (“it is / it is not a...”). They use simple language. They do not identify the properties of geometric figures.



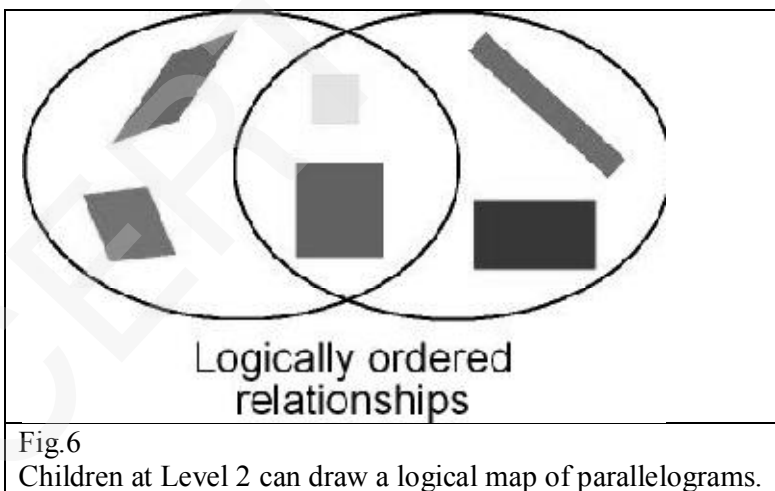
Level 1 Analysis (Description)

At this level pupils (students) start analyzing and naming properties of geometric figures. They do not see relationships between properties, they think all properties are important (= there is no difference between necessary and sufficient properties). They do not see a need for proof of facts discovered empirically. They can measure, fold and cut paper, use geometric software etc.



Level 2 Abstraction (Informal deduction or Ordering or Relational)

At this level pupils or students perceive relationships between properties and figures. They create meaningful definitions. They are able to give simple arguments to justify their reasoning. They can draw logical maps and diagrams. They use sketches, grid paper, geometric SW.



Pierre van Hiele wrote: “My experience as a teacher of geometry convinces me that all too often, students have not yet achieved this level of informal deduction. Consequently, they are not successful in their study of the kind of geometry that Euclid created, which involves formal deduction.”

Level 3 Deduction (Formal deduction)

At this level students can give deductive geometric proofs. They are able to differentiate between necessary and sufficient conditions. They identify which properties are implied by others. They understand the role of definitions, theorems, axioms and proofs.

Level 4 Rigor

At this level students understand the way how mathematical systems are established. They are able to use all types of proofs.


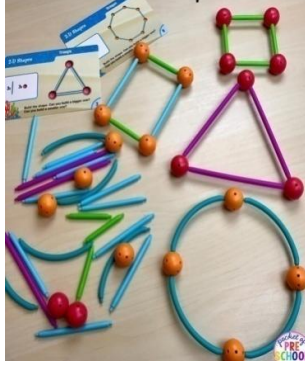






Van Hieles believed that cognitive progress in geometry can be accelerated by instruction. They progress from one level to the next one is more dependent upon instruction than on age or maturity. They gave clear explanations of how the teacher should proceed to guide students from one level to the next. However, this process takes tens of hours.

8.6 IDENTIFYING AND CREATING SITUATIONS/ ACTIVITIES TO TEACH SHAPES, PATTERNS, POSITIONS

There are many ways shapes can be introduces to children. Let us see some of the activities/ play way methods through which we can help them build on these concepts.

Introducing shapes through playing, manipulation and construction activities

In every early childhood classes it is a must to teach about two-dimensional shapes. The activities can be planned with Lego blocks, play dough, broom sticks, match sticks, geoboards, paper cut outs, tooth picks, card boards etc. You can plan a shape of the week activity or you can spread your activities throughout the academic year. These activities can be planned for any type of classroom and teaching styles.

			
<p>Fig.7 Using solid shape blocks to create patterns</p>	<p>Fig.8 Using manipulatives to create shapes</p>	<p>Fig.9 Using toothpicks and play dough to create shapes</p>	<p>Fig.10 Using Lego blocks to make construction activities</p>
			
<p>Fig.11 Geoboard for free play and shape creation</p>	<p>Fig.12 Shape sorters to sort shapes</p>	<p>Fig.13 Play dough or coloured atta to make shapes</p>	<p>Fig.14 Colour paper cut outs for making shape collages</p>

Importance of manipulation activities:

Learning about shapes is important but children should be able to have control of their world. This is possible when children play in groups with their peers; work on collaborative tasks by using sufficient resources. In this process children learn to manipulative by turning objects, screwing/unscrewing items, moving, ordering the items, and checking if the object fits.

Manipulative items help children:

- In decision making
- To understand different shapes, sizes, lengths, weights, heights
- To understand order, patterns, colours matches, sequences, and to make comparisons
- To know about cause and effect relationship
- To develop attention, concentration and organized play

Equipment to be provided by a teacher / school for manipulative play

- Threading materials such as beads, buttons, threads, reels, strings,
- Screwing materials such as nuts, bolts, screws, plastic containers with lids
- Sorting and matching items for colour, shape and size
- Stacking blocks such as interlocking blocks Mega Blocks

Show and tell method to learn about shapes:

Shapes can be reinforced by show and tell or show and share method. This is generally the first time opportunity given to the little ones to stand up and speak in front of a small group. They can pick any shape of their choice or compare shapes, give description of the given shape and relate to day-to-day examples. This method helps in

- Building confidence
- Develops interest for the math as a subject
- Improve on their subject awareness
- Develops pride and self esteem
- Promotes turn taking
- Child gets to be the centre of attention
- Interact with peers and teacher

We also find few limitations from this method

- Required planning
- Time taking
- All children may not be confident and lose hope
- May not be able to define shapes or compare shapes
- May not be able to use appropriate math words

These limitations can be overridden easily with the assistance provided by the teacher.

Plan game through which 2D or 3D shapes can be taught to kids.

Activities about shapes and patterns:

<p>Make your own patterns.</p>		
<p>Fig.15 Activities on Patterns</p>	<p>Fig.16 Matching 2-D shapes</p>	<p>Fig.17 Counting sides and corners in a shape</p>
	<p>Write the name of each shape. Use the words in the Word Box.</p> <p>Word Box sphere prism cone cube cylinder pyramid</p>	
<p>Fig.18 Circle the polygon with same number of sides and Circle the polygon with different number of sides</p>	<p>Fig.19 Naming the shape from the word box</p>	<p>Fig.20 Activity related to 3-D Shapes</p>

Math Positional Words:

Understanding the position of the object and its opposite is related to positional language. For example: *inside and outside, in front or behind, between and beside; above, on and below*. All these words are necessary for future learning.




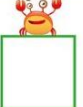
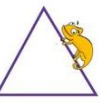










<p>Which picture shows the duck inside the circle ?</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px;"> <input type="checkbox"/>  </div> <div style="border: 1px solid black; padding: 5px;"> <input type="checkbox"/>  </div> </div> <p>Which picture shows the crab outside the square ?</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px;"> <input type="checkbox"/>  </div> <div style="border: 1px solid black; padding: 5px;"> <input type="checkbox"/>  </div> </div> <p>Which picture shows the chameleon outside the triangle ?</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px;"> <input type="checkbox"/>  </div> <div style="border: 1px solid black; padding: 5px;"> <input type="checkbox"/>  </div> </div>	<p>Circle the object that is above the table.</p> <div style="display: flex; justify-content: space-around;">    </div> <p>Circle the object that is below the table.</p> <div style="display: flex; justify-content: space-around;">    </div> <p>Circle the object that is above the table.</p> <div style="display: flex; justify-content: space-around;">    </div>
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Fig.21
Inside-Outside position

Fig.22
Above – Below Position
Can also be used for On, Beside and between

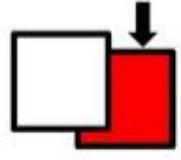
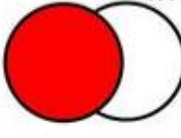



























<div style="border: 1px solid black; padding: 10px; margin-bottom: 10px;">  <p style="text-align: right;">Behind</p> </div> <div style="border: 1px solid black; padding: 10px;">  <p style="text-align: right;">In front of</p> </div>	<p>Look at the shapes and answer the questions.</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table> <ul style="list-style-type: none"> circle hexagon diamond pentagon rectangle square star triangle <p>Which shape is ...</p> <p>undersneath the circle? _____</p> <p>to the left of the triangle? _____</p> <p>above the hexagon? _____</p> <p>below the pentagon? _____</p> <p>between the rectangle and the diamond? _____</p> <p>diagonally above the empty space? _____</p> <p>beside the diamond? _____</p> <p>on top of the diamond? _____</p> <p>between the triangle and the star? _____</p> <p>on the right-hand end of the top row? _____</p> <p>in the centre of the grid? _____</p> <p>in the top left-hand corner? _____</p>									
										
										
										

Fig. 23
In Front of and Behind Position

Fig.24
Shape with its position

8.7 SUM UP:

Learning of shapes helps children in identifying and organizing visual information. We find shapes everywhere in our classroom, could be our notebooks, pencils, erasers, tube lights, tiffin boxes, water bottles etc. We have seen several activities to teach shapes, patterns and positions. These help children in enhancing their communication skills and vocabulary. They pave way for greater creative thoughts and help children in transiting from stick figures to more detail and creative drawings, learning alphabets and numbers; and crafts. Improves on their sorting skills and extends ability to think about the attributes of objects and related items. Truly they facilitate in developing logical thinking among children.

8.8 QUESTIONS:

1. What is a polygon? Why is it necessary to learn about these polygons?
2. Take any four 3D shapes and plan activities to teach these to your classroom kids.
3. State the five levels of thinking as per Van Hiele in learning geometry.
4. Explain the importance of manipulative play.

UNIT 9 - DATA HANDLING

9.0 OBJECTIVES

After completing this unit as a preschool teacher trainee, you will be able:

- To understand data handling
- To understand the importance of data handling in modern day life
- To explain different stages in data handling cycle
- To describe the knowledge that children need to possess about data
- To apply data representation (diagrams, charts, graphs) strategies in the classroom

9.1 INTRODUCTION

A child starts gathering information even before birth. We notice that infants too can store and make out their mother's voice and touch. A just born baby is able to distinguish their mother to other people. The reason is, the baby has already recorded the data and processed the data i.e. mother's voice through intonation, stress and rhythm into an organized form that is information. This processing of data is inherent in our system and can be developed with the growing years of the child.

Data is everywhere and we notice that small children are good at grasping and learning quickly. They mostly make use of data and unconsciously apply on several occasions that brings astonishment to many of us as teachers.

Children utilize several play items while they are playing. They utilize pebbles, fruits, paints, shapes, dumbbells, alphabets, numbers, sweets, chocolates, toys and after they finish playing, we notice that they are good at organizing them back into their boxes in most cases. This is because their brain is processing the data to produce information. All this comes under organizing the data and in other words called as data handling.

9.2 WHAT IS DATA HANDLING?

In early years of schooling data handling means being able to gather the data, record the processed data as information and then present to others in a meaningful way to solve a problem.

Handling data is a crucial aspect of mathematics which relates to collecting, representing and analysing data in order to solve a particular problem or question (Montague-Smith & Price, 2012; Reys, Lindquist, Lambdin, Smith, Rogers, Falle, Frid & Bennett, 2012).

Data handling allows children to make sense of information, to identify patterns and trends and to predict and plan for the future. (Griffiths, 2001).

9.3 IMPORTANCE OF DATA HANDLING IN MODERN LIFE & RELATING WITH DAILY LIFE ACTIVITIES

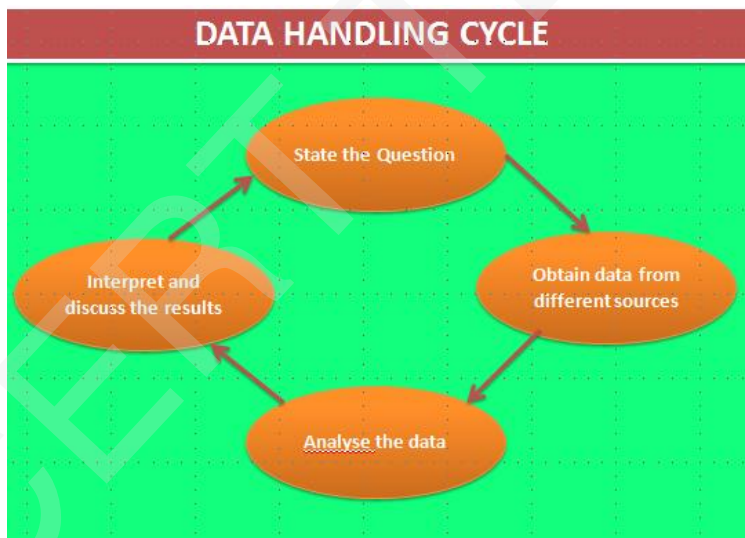
A child's natural ability is being able to observe the world around them and they have the ability to basically sort the data to some extent. As a teacher it is our responsibility in assisting the children in recognizing the dominant data during collection and lend a supporting hand to them in order to process. While creating classroom activities, we need to keep in mind how data handling can help in a child's development in learning.

Data handling is of utmost importance in day to day life as well as in today's modern world. Here are some of the instances where we can see data handling:

- In schools – attendance of the students or progress made by the child over a term
- Doctor's clinic – to keep track of vaccinations given to a child
- At home – to keep track of monthly expenses
- Among children – finding their favourite chocolates/ biscuits
- In a garden/park – counting different variety of plants/insects

Explore some more instances of data handling in day-to-day life.

9.4 PROCESS OF DATA HANDLING & IT'S CYCLE



There are many essentials to consider at every stage of the cycle, the process comes up with following questions:

- ✓ Why to collect data?
- ✓ What kind of data needs to be collected?
- ✓ How are we willing to collect, organise and represent the data?
- ✓ How reliable is the data?
- ✓ Which is the best way to represent the data, either graph or chart?
- ✓ What 'story' can we say from this data?

As a teacher it is vital that we understand data handling cycle to ensure a balance of activities to be provided that include collection of data, organization of data based on analysis, representation of data, finally interpreting and discussing results.

9.5 KNOWLEDGE ABOUT DATA TO CHILDREN

At the basic stage the information that we collect knowingly about the world is called data. To make this information more meaningful we have to classify, sort, organise, represent and finally interpret the data.

Now let see what children need to know:

1. Concepts related to data
2. Understand the question
3. Analyse and represent the data
4. Interpret and discuss the results

9.5.1 Concepts related to data

To make use of data, children should be able to:

- **Recognize differences:** The children should be taught to classify the given objects based on colour, size, weight, height, utility, appearance, also should be provided with activities that include human distinctiveness such as height, opinions and roles.
- **Categorise information:** Here some of the examples could be related to taste (favourite food such as chocolate or ice-cream or biscuit), form (Cycle, Bike, Cars and all kinds of transportation but have different forms), shape (triangles, rectangles, squares, circles) and many more.
- **Sort information:** Based on climatic conditions of seasons or based on division of individuals in different regions etc.

9.5.2 Understand the question

Understanding the question and its context is very important for a child. It will be difficult to find out what kind of data is required and needs to be collected from the child's standpoint. As a teacher you need to help them to bear in mind the following aspects:

- What is the sample?
Suppose Section 'C' students from a class are going on a picnic. Their dress code is given as red. Assume a child comes in green colour dress, then the question arises in the mind of the child as to whether children in any other colour, apart from red dress are allowed to go for the picnic or not? So children need to think beyond the current state in order to know whether this can be included in the sample or not. Right information/direction at this point needs to be given by the teacher.
- What needs to be measured—the variations and possibilities available? What if some of the kids go a movie every weekend, some of the kids go to the movie rarely, and some of the kids never go to a movie. Kids may find it hard to think about various options under several circumstances.
- Likewise how frequently the data needs to be collected, during what time the data has to be collected, several questions arise in the minds of the children where the teachers assistance is required to bring in clarity.

9.5.3 Analyse and represent data

After the data has been collected it has to be put in a meaning format (*also known as tabulation of data*) so that it aids in answering the question and arriving at the solution to the given problem. There are different ways the data can be analysed and represented. It could be in the form of bar graphs, tally charts, pictographs etc. *We will see in detail in 9.6.*

9.5.4 Interpret and discuss the results

Once the question is rightly understood from different contexts then interpreting the findings within the context of the given question paves way to solution. These thoughts on how to interpret over the years helps in building upon child's scientific attitude.

9.6 REPRESENTING VARIABILITY OF DATA USING DIAGRAMS, CHARTS AND GRAPHS

We will see different charts, graphs to represent data. There is no universal conformity on what comprises a diagram, graph or chart. Frequently we have seen several teaching materials might use diverse terms to refer to the same thing. It is imperative to be steady when using these terms in order to avoid needless confusion. We will be learning about how to introduce the following into your classrooms:

- ✓ Table and Tally Chart
- ✓ Pictograph / Pictogram
- ✓ Bar Graph / Bar Chart

9.6.1 Table / Tally chart:

Definition:

A table is a organised way of representing data(information) in rows and columns.

Illustration:

A class of 20 students may be asked to as to which chocolate (Milk-M or White-W or Dark-D) they like the most.

The responses received from students are given in a table.

W	W	M	D
W	D	W	W
M	D	W	W
D	W	W	M
W	M	M	W

The data has been collected and placed in a table. But can we interpret anything from this? Not easy

Now let us draw a tally chart from the above responses:

Tally Chart:

Fruit name	Tally	Students liking chocolate
Milk Chocolate	/	5
White Chocolate	/ /	11
Dark Chocolate		4
Total children		20

A tally marks are a quick way to arrange the frequencies or occurrences. They are grouped into 5 for easy counting. Every occurrence is marked as a vertical line. After every 4 occurrences the 5th fifth occurrence is represented by a diagonal line across the previous four for a simpler counting. This needs to be clearly explained to the children as most of them get confused at the beginning. With regular practice through different activities the child generally masters at drawing tally marks and tables.

Now from the tally chart and the table, it is easy to interpret that 5 students like milk chocolate, 11 students like white chocolate and 4 students like dark chocolate.

General Information:

The data that is frequently collected is generally organised and presented in the form of a table. Later the data can then be moved to a suitable chart or graph type. Table will provide information under two headings. Examples of tables are tally tables, timetables, distance tables and catalogues etc.

Key Highlights

- A table should be in logical order
- The data should be in rows and columns
- Rows and columns should have titles and units (if needed)

Teaching activities / ideas

1. Children may given worksheets to practice tally marks.

Which tally marks show 13?



Which tally marks show 15?



Which tally marks show 17?



Which tally marks show 23?



2. Students may be asked to pick different shapes (circles, rectangles, triangles, stars) from a box. They may be asked to arrange similar shapes in a row, similarly for all the other shapes. Then ask them to make a tally chart and state the frequencies.

9.6.2 Pictograph / Pictogram:






Definition:

A pictograph uses pictures or images or symbols to represent the frequency of the items. Even part of the picture or image is taken into consideration to give an approximate proportion of the given number.

Illustration 1:

Table

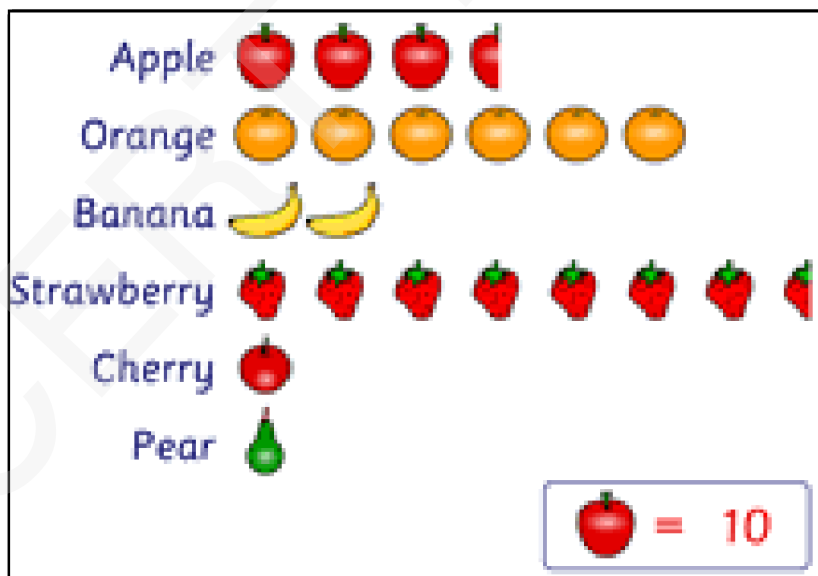
Fruit name	No. of children who like this as their favourite fruit
Pear	7
Watermelon	1
Orange	1
Apple	1
Banana	1
Total children	11

FRUIT	NUMBER OF CHILDREN WHO CHOSE IT
PEAR	
WATERMELON	
ORANGE	
APPLE	
BANANA	

The above pictogram shows the most favourite fruit among children.

Illustration 2:

Fruit name	No. of fruits eaten by the children
Apple	3½
Orange	6
Banana	2
Strawberry	8
Cherry	1
Pear	1
Total children	21½



The above picture shows how even proportions can be represented in a pictograph.

General Information:

A pictograph is an easy way to represent data through pictures or images or symbols. The selected image can be used to correspond to any value, and this must be specified in a 'key'. The child should be aware that representation is not accurate and are mostly rounded to the nearest number.

Key Highlights

- Every picture to represent numerical data
- A picture can symbolize any number
- Pictograph must possess a title
- The selected pictures have to be of the same size and must be uniformly spaced
- Part of a picture($\frac{1}{2}$, $\frac{1}{4}$) can represent as a part of the whole picture
- Pictures in the pictograph should be arranged horizontally or vertically
- Key is to be included to show what the images mean or represent in numerical terms

Teaching activities / ideas

1. Child may be asked to find the number of glasses of water each family members drinks at home every day. After child comes back to school with data, the child may be assisted in drawing a pictograph.
2. Children may be asked to observe from their window and to take a count of vehicles that go on the street. A table and a pictogram may be drawn by the child.
3. A worksheet of similar representation can be prepared and asked to be filled by the child.

Number of pets

cat bird fish horse dog rabbit

Pets

How many pets?

Draw the pet that matches the number.

6 2 1

Number of shapes

triangle square oval rectangle circle star

Shapes

How many shapes?

Which shape matches each number?

4 0 3

9.6.2 Bar Graph / Bar Chart:

Definition:

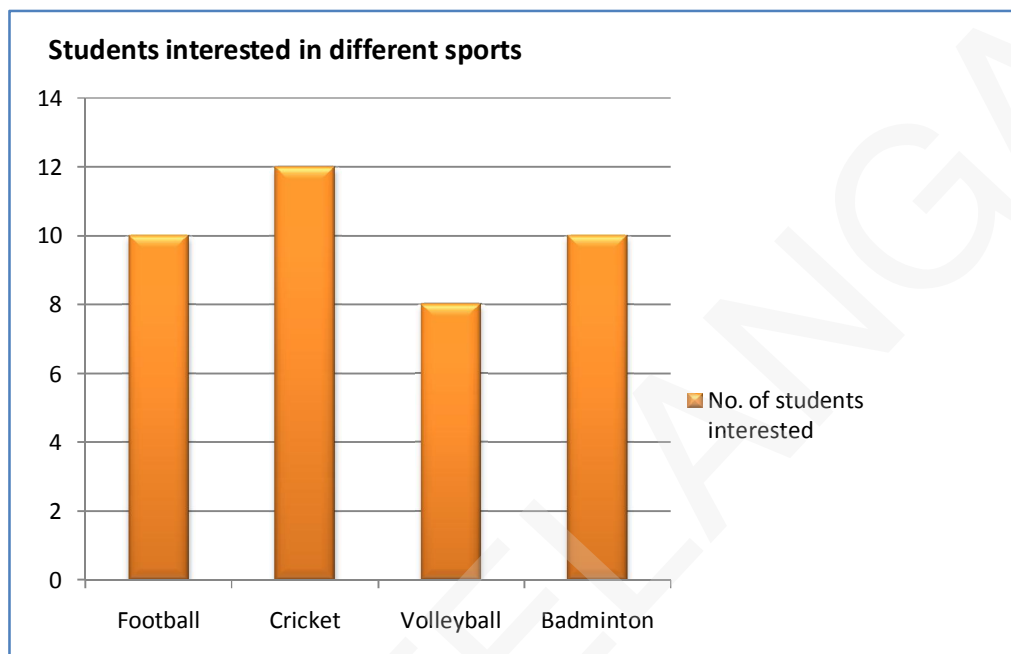
A Bar graph represents data in which different frequencies / occurrences are represented using vertical or horizontal bars of equal width. Frequencies that are higher have longer length of the bar.

Illustration 1:

Table

Sport	No. of students interested
Football	10
Cricket	12

Volleyball	8
Badminton	10
Total students	40



Bar Graph showing students of a class interested in different sports

Interpretation: It may be noted that there are 40 students. Students interested in football are 10, students interested in cricket are 12, students interested in volleyball are 8 and students interested in badminton are 10.

General Information:

They are simple to draw and clearly help in demonstration and comparison. There are several varieties and styles in which bar charts can be drawn. Simple style may be taught to children at the beginning.

Key Highlights

- A bar graph should have a title
- Should be drawn on a graph paper or a squared sheet
- Each bar should be neatly labeled
- Bar should not be split into blocks
- Choose a most appropriate scale based on the data

Teaching activities / ideas

1. A class of students may be asked their favourite flavour of ice-cream and then a tally table with frequencies to be created. Using the table, a bar graph can be represented.
2. A worksheet containing a bar graph is to be given to students and may be asked to interpret the data.
3. A squared sheet with frequencies to be given to students. The students may be asked to colour the squares based on the frequencies.

Explore different varieties of bar graphs to represent various kinds of data that you find in your daily life.

9.7 SUM UP:

Data handling is a practical topic that can be used across the curriculum. It plays an important role in math education as it encompasses real world situations. It helps in building critical thinking among the children, helps them to learn range of skills that include exploration, gathering data based on concepts and understanding, interpreting/ examining and analyzing data and finally presenting the data in a meaningful way.

As a teacher you have a great potential to bring in enthusiasm among children and help in brightening up the classroom through various colourful displays of tables, tally charts, bar graphs, pictograms.

Explore technology based apps to represent data and use them in your classroom.

9.8 QUESTIONS:

1. What is data handling? Explain the importance of data handling in modern day life.
2. Explain in detail the kind of knowledge students should possess regarding data.
3. Explain different ways to represent data with examples.
4. Create a worksheet for your students to get a better grasp of tally chart.
5. What are the essentials to be considered at every stage of the data handling cycle?

UNIT 10 - PLANNING AND CONDUCTING MATH EXPERIENCES IN THE CLASSROOM AND ASSESSING PROGRESS (PRACTICAL)

10.0 OBJECTIVES

After completing this unit as a preschool teacher trainee, you will be able to:

- Understand the importance of free play in supporting math concepts in your classroom
- Demonstrate on conducting dialogue and lead discussions to foster learning among children
- Differentiate between regular classroom and multi-graded classroom and handle variability
- Plan on teaching mathematical language and concepts in an integral way
- Develop material for effective instruction by creating year plan, unit plan and lesson plan

10.1 INTRODUCTION

Early childhood knowledge and expertise can have an everlasting impact on children's later life. Strengthening the areas of development and learning with appropriate practices can support child's cognitive development, social skills, and self regulation abilities. Children benefit from a strong curriculum in math when the curriculum is delivered with age appropriate playful pedagogy lead by right discussions.

The challenge is how to plan and implement these in the classrooms to foster learning from the early age so that they reach their mathematical potential. We will see some of the strategies where we can support free play to teach math concepts.

10.2 SUPPORTING FREE PLAY AND BE AWARE OF THE OPPORTUNITIES THAT ARISE TO SUPPORT MATHS CONCEPTS

Kids during their early years are naturally curious and from the researches over the years it is imperative that the best time to introduce mathematical concepts to the little ones is at the time when their brain is fast developing. Learning through play, songs, games, toys and daily activities can be easy and fun. The experiential learning of Math during childhood promotes reasoning skills and critical thinking which lays the foundation for their future schooling.

Learning through play can be categorized into

- Free play – It is initiated by the child where the s/he learns through engaging in recreation, manipulating objects, working together with his/her peers or teachers.
- Guided play – This is done either by a teacher or can be by a parent. It is a directed activity to learn a concept.

Seo and Ginsberg reviewed videotapes of four and five-year-old children as they played for fifteen-minutes to determine the types of mathematics that occurred organically in everyday play. Six categories of mathematical content emerged: classification (grouping or sorting by attribute), magnitude (comparing the size of objects, such as a tower built of blocks), enumeration (saying number words, counting, subtilizing, or reading/writing numerals), dynamics (putting things together or taking things apart), patterns and shape (for example, making a necklace out of beads with a pattern), and spatial location (describing a direction or location).

Using playful strategies that are child directed and guided in our classroom, we get the opportunity to deliver rich mathematical learning experiences to our children. Here are some of the video links that help us understand how to teach math through free play.

Click on the link to see the YouTube video supporting free play and learning math:

Kindergarten Free Play. https://www.youtube.com/watch?v=-VK3f07kbpo duration (1:01)

Importance of Play. https://www.youtube.com/watch?v=vnH4Ijen7OI duration (5:20)

From the above video we can plan how children can use free play to in Math to

- Sort items / classify in the boxes, containers
- Compare objects
- Enumerate by counting and recognising objects
- Explore how to put things together, take them apart, flip items,
- Describe the objects and toys
- Work on spatial relationships
- Study the patterns and shapes

10.3 CONDUCTING DIALOGUE AND LEAD DISCUSSION

An interaction or a conversational talk between two/more people or in a group is called as a dialogue. Dialogue plays an important role in teaching and learning process as it paves way for exploration on a particular subject or resolving a given problem. It involves exchange of thoughts and perceptions. In a Math class we need to develop dialogue among the peer group and teacher to encourage collaborative learning culture for frequently exchanging ideas.

Types of dialogue that we can have in our classroom are:

1. Oral questioning
2. Question and Answer
3. Think / Pair / Share
4. Discussions
5. Debates (Formal / informal)
6. Presentations

Framing questions from the teacher's end is vital to promote discussion in the class. How can we do this? Here are some of the questions that can be used in such a scenario by the teacher in the classroom:

- How can I solve this problem?
- Which method have you chosen?
- Why do you think this is the best method?
- Is there any other strategy to solve?
- Can we think a bit differently, pls come out with your ideas.
- What would happen if....?
- What do you think about it...?
- Tell me about?
- In what way are the(similar / equal / different)?
- How can we do...?
- How did you do...?
- In what way...?
- Describe your strategy...
- Explain in detail...
- How did you do and when?
- How do you which one?

We will see some of YouTube videos on how to have a dialogue/ talk in our classrooms.

Click on the links to see the YouTube videos to know about leading discussion in the classroom:

Math Talk Sample Lesson. https://www.youtube.com/watch?v=yR69-izSIiE duration (18:11)

Using Math Talk to Support Learning. <https://www.youtube.com/watch?v=TLmm3U0eYX4>
duration (5:16)

Building Mathematical Competencies in Early Childhood.
<https://www.youtube.com/watch?v=BMoF-hiH3J8&t=27s> duration (4:59)

There are numerous benefits that we can get through Math talk based on the above videos:

- Enables kids to become active questioners
- Participate in the discussion
- Help them to lead discussion in which children can express their method of solving a problem
- Receive help from peers or teachers
- Identify errors and discuss on how to correct and solve
- Reason without fear of being right or wrong
- Stimulate engagement in the classroom and in the community
- Get new perspectives to think on the problem by taking cues from other children
- Raise voice again and take other peers into discussion
- Frame questions and think out of the box
- Engenders flexible thought process
- Clarify on student's own thinking and explain the same to others / peers
- Develop math language
- Enables to build interest, confidence and self-esteem
- Improve the performance of the child by motivation

Apart from the above list, there are many more benefits that can be derived through dialogue and discussion.

List down few more of them.

10.4 HANDLING VARIABILITY IN THE MULTIGRADED CLASSROOM

A classroom consists of different set of students, may not be homogenous all the time. Children can differ in their IQ, social background, societal setup, religious influences and many other factors. We find lots of variability or differences among children of same class and multi-graded classroom. In a multi-graded classroom we find students from different grades are being handled by a single teacher. This generally happens in remote or rural schools with limited size of the student population.

Click on the links to see the YouTube videos to know about handling classroom variability in multi-graded classroom and regular classrooms:

Teacher tips- How we do math stations. https://www.youtube.com/watch?v=GXV7F1rmumw duration (9:39)
Multi-grade Teaching. https://www.youtube.com/watch?v=YF-m_VNckc duration (3:29)
Management of Multi-Grade and Multi-Level Teaching. https://www.youtube.com/watch?v=zC7LS_bNs4 duration (21:01)

Based on **Teaching and Learning in the Multigrade Classroom**, Miller (1991) has identified six instructional dimensions affecting successful multi-grade teaching. They are:

- Classroom organization
- Classroom management and discipline
- Instructional organization and curriculum
- Instructional delivery and grouping
- Self-directed learning
- Peer tutoring

In multi-grade or multi-class settings of instructional organization, teachers should spend more time on organizing and planning of instruction in order to maintain the continuity of students' learning, utilization of available resources in the classroom and promoting interactive, participatory and child centered learning in the classroom.

How to teach counting for 3 year olds and data handling for 5 year olds in a multi-class settings?

10.5 TEACHING MATHEMATICAL LANGUAGE & CONCEPTS IN AN INTEGRAL WAY

Teaching of math should be woven throughout the day's activities from the childhood, since learning math in real life contexts with experiences nurtures love for the subject for life. A routine in math activities involve learning numbers, ability to count, addition and subtraction, spatial relationships, data handling etc. that can teach them measurement and categorization. How can we teach these? The answer is by playing games, building toys, working on puzzles, engaging through blocks, fun through physical activities that help them to build math skills in a engaging way in real life context.

We will see integral ways in which math can be taken to our children.

Click on the links to see the YouTube videos to know about teaching math in a natural setting:

Math & Movement. <https://www.youtube.com/watch?v=9HdDRh0Ohvk> duration (3:05)

RPS LKG Maths Activity. https://www.youtube.com/watch?v=1DekV_DeVXM duration (3:58)

Here are some ideas where mathematical language and concepts can be taught in an integral way:

- **Realistic math:** Using handy or household things to practice addition and subtraction. If you have a basket of 10 biscuits and if your friend eats 4 of them, how many will be left?
- **Making patterns:** Use cereal to make jewellery like neck pieces, bracelets or sandwiches can be layered with cut veggies, or crackers to create simple patterns.
- **Building things:** Uses legos, blocks and other toy stuff to create and construct tall buildings, houses. This helps the child to come up with counting, comparison, understand shapes and create patterns
- **Nature walk:** Step outside the classroom, go around a garden to understand words like big, small, large, tiny, up, down; positional math; sizes, shapes; colours etc.
- **Data handling:** Take a poll to find which chocolate is loved by kids (Five Star, Cadbury, Perk, Galaxy) and draw a bar graph to teach the concept for analyzing data.

Integral way of teaching mathematics is not like the traditional method, where the focus is on learning core mathematical concepts. Here math is taught in real life setting in a very casual way. It is a psychologically sound system of learning based on the principles of learning by doing. Children are able to understand the concepts and facts clearly as it involves fun, practical work through careful observation and independent thinking.

How can we teach parts and whole to children in an integral way?

10.6 PLANNING FOR EFFECTIVE INSTRUCTION

Planning in education is very important because it makes sure of achieving success. During the planning phase we consider significant aspects, conditions, social settings, constraints and other related factors. We also work in a systematic fashion to achieve potential objectives and plan our goals.

Planning of instruction is generally planned at three levels:

Level I	Planning the teaching and learning for the full year	Year Plan
Level II	Planning the teaching and learning of a unit	Unit Plan
Level III	Planning the instruction of a topic	Lesson Plan

10.6.1 NEED & SIGNIFICANCE OF YEAR PLAN:

A teacher generally prepares a year plan once at the beginning of the academic course. As teachers we need to tentatively outline, give the topics to be covered during various months as per the academic calendar and the classes allotted. The following points should be kept in mind while preparing the year plan:

- Objectives that need to be achieved during that academic year
- Content / Topics / Units to be covered based on the given syllabus
- Working days and number of periods allotted to cover each unit
- Available material and resources to be considered
- Capability and psychological aspects of the students to be considered
- Prioritize among the objectives

Format for a year plan:

Name of the school: _____

Name of the teacher: _____

Class and Section: _____

Subject: _____

Academic year: _____

S · N o ·	Unit	Objectives							Periods			Calendar Months
		Knowledge	Understanding	Application	Skill	Interest	Attitude	Appreciation	Instruction	Activities	Evaluation	

10.6.2 NEED & SIGNIFICANCE OF UNIT PLAN:

A unit can be termed as a block of work, or a chapter in the textbook or a sub division of well organised subject matter.

Definition of a Unit:

Preston defined a unit is a large block of related subject matter as can be over-viewed by the learner.

Samford defined a unit as an outline of carefully selected subject matter which has been isolated because of its relationship to pupil's needs and interests.

Unit planning is a part of year planning and is in the middle level between year planning and lesson planning.

Format for a unit plan:

Name of the school: _____

Name of the teacher: _____

Class and Section: _____

Subject: _____

Unit: _____

S.No.	Subunit Name	Objectives / specifications	Scope of teaching	Teaching & learning activities	Method to be followed	Teaching & learning material required	No. of Periods required

Procedure to be followed for writing a unit plan:

- Unit to be chunked into subunits in a meaningful way
- Each subunit has to be divided into lessons
- Based on the number of lessons, instructional periods, methodology, activities, teaching and learning material much be planned
- Plan for variety of learning experiences

A good teaching requires necessary planning and implementation for successful achievement of objectives.

10.6.3 NEED & SIGNIFICANCE OF LESSON PLAN:

A great planning is required for day-to-day work. The teacher needs to plan considering the time and effort in achieving the objectives. For this a meticulous lesson plan needs to be created.

Definition of a Lesson Plan:

Davies defined lesson plan is an arrangement of learning, planning, organizing, and controlling by a teacher.

Good defined a lesson plan is an outline of the important points of a lesson arranged in the order in which they are to be presented to students by the teacher.

Guidelines for preparing the lesson plan:

- Select a suitable topic and write down the objectives (general & specific) / specifications clearly
- Should give a detailed outline of the various steps that a teacher plans to take in the class
- Should contain oral and preferably written form of learning
- Need to begin by considering the previous knowledge and experiences of the learner
- Provision should be made to arouse interest and curiosity among the learners
- Should reveal the development of the topic
- A list of audio visual aids needs to be specified and should clearly state how to use them

- Activities should be suggested to work towards individual differences
- Should contain the questions to be asked, illustrations to be stated and assignment to be completed
- Consolidation and recapitulation of ideas to be mentioned
- Clearly cite the references
- Supposed to stimulate reflective thinking
- Should provide intrinsic and extrinsic motivation

Steps in preparing a lesson plan:

John Fedrik Herbart, a German philosopher and educationalist (1776-1841) advocated pedagogy based on lesson planning. Herbartian approach to lesson planning involves the following steps:

- Preparation
- Presentation
- Association
- Generalisation
- Application
- Recapitulation

Alternatives to Herbatian steps:

There are several other alternatives to Herbatian steps that we can also follow:

1	Introductory Phase	Testing of previous knowledge and experiences
		Motivation
		Declaration of the topic
2	Presentation Phase	
3	Culminating Phase	Conclusion
		Recapitulation
		Homework / Assignment

Click on the links to see the YouTube videos on how to write a lesson plan:

Maths Lesson Plan (Big & Small). https://www.youtube.com/watch?v=t_nVDce_sx0&t=8s. duration (2:21)

Lesson plan for maths. <https://www.youtube.com/watch?v=Woka-ArrJIw> duration (2:34)

Based on the above discussion, you can plan lessons on various other topics such as ‘Number talk’, ‘Space talk’, Number sense, data handling etc.

10.7 SUM UP:

In this unit we have learnt through videos on the following:

- How to support free play and look for opportunities to teach math concepts
- How to conduct dialogue and lead discussions, types of dialogues, questions that can be posed to promote discussion among children, benefits that we get from math talk
- Handling variability in a multi-graded classroom
- Teaching mathematical language and concepts in an integral way and some ideas that we can use in our classroom
- Planning for effective instruction – year plan, unit plan and lesson plan

10.8 QUESTIONS:

1. Discuss in detail on how to support free play to teach math concepts.
2. Explain how children can benefit from conducting dialogue.
3. State how to handle variability in a multi-graded classroom
4. How to teach math concepts in an integral way?
5. Create a unit plan by choosing a topic of your choice related to math.
6. Design a lesson plan on the topic place value in math.
